Conditional Restriction

Recall an observation from Intensional Semantics 101: in conditional consequents, modals quantify only over worlds where the antecedent holds.[1] Take:

(1) If that is a dinosaur, then we must be in Jurassic Park.

Here the domain of "must" is restricted to antecedent worlds. Focus on the accessible worlds where that is a dinosaur: in all *those* worlds, we are in Jurassic Park.

Such examples reoriented the linguistic semantics of conditionals. On the restrictor semantics for if-clauses, introduced by Lewis (1975) and championed by Kratzer (1981, 2012), if-clauses simply restrict the domains of modals; the apparent modal flavour of "if" derives entirely from the presence of modals. For almost as long as the conditional has been studied, the contribution of "if" has apparently been fundamentally misunderstood.[2]

This phenomenon of conditional restriction has not been much studied from a *logical* point of view. I show there are important, fruitful questions here. Can we characterise the restrictor data as axiomatic principles? And if so, what kind of conditional logic would allow us to derive them? I aim to answer both questions here. I propose a conditional logic, which I call R, as an axiomatic regimentation of the restrictor view of conditionals. I show that this logic has theorems that axiomatically capture the phenomenon of conditional restriction.

Axiomatisation offers a roadmap for any kind of theory to capture conditional restriction. Views which validate this logic will be ones where conditional antecedents have a restricting effect. One important lesson will be that, despite its considerable impact in the lingusitics literature, Kratzer's pure restrictor theory is not forced on us by the basic data.

Here is the plan. I start with what I call the basic modals, possibility and necessity modals of epistemic or historical flavour, and regiment the restrictor data in the form of a number of axiomatic principles. I outline the R logic and show how to derive the restrictor data as theorems. I offer a competitor to the pure restrictor view, a *restricting operator* semantics

[1] Note this paper will focus entirely on the restricting effects of *indicative* conditionals.

[2] The restrictor view is now extremely widespread in linguistic semantics; see Heim (1982) and von Fintel (1994) for further important discussions. For discussion of this view in the philosophical literature, see Khoo (2011), Rothschild (2021), Mandelkern (2018b, 2021), Cariani (2021), Santorio (2022), Ciardelli (2021a,b).

where "if" contributes a kind of variably strict semantics and which validates the R logic. I compare it to the pure restrictor view, arguing it has an important advantage in avoiding appeal to covert modals. I then consider the case of deontic conditionals, showing how to regiment the restrictor data for deontics, how to derive it axiomatically in an extension of R and show how the restricting operator view might validate that extended logic. I also show how such a view might be extended to capture adverbial conditionals.

1 Basic Modals

Let's start with a textbook case of conditionals restricting epistemic modals:

> *A drive through MA.* Our friends have been driving in the Massachusetts hinterlands ... relying entirely on an old-fashioned map. They've just passed through a little town with an iconic New England church and are looking on the map to try to figure out where they are. They have concluded that they are either on Route 117 or on Route 62. There are two plausible candidate towns on Route 117 (Maynard and Stow) and just one on Route 62 (Hudson). (von Fintel, Kai and Irene Heim (2021))

Von Fintel and Heim's first data point concerns conditionals with necessity modal consequents. Our friends say:

(2) If we are on route 62, then we must be in Hudson.

(2) feels equivalent to an unconditional modal claim, namely that, in all of the epistemically possible worlds where we are on route 62, then we are in Hudson.

This intuition about truth-conditions can be captured with a logical principle. In general, in all the epistemically possible worlds where A is true, B is also true just in case in all the epistemically possible worlds $A \supset B$ is true. Thus, we should aim to validate the following:

Boxiness. $(A > \Box_E C) \leftrightarrow \Box_E(A \supset C)$

This principle states necessary and sufficient conditions: $A > \Box_E C$ is true iff all epistemically possible A-worlds are C-worlds.[3]

A second data point concerns conditionals with epistemic possibility consequents. In the case above, von Fintel and Heim note that our friends can say (3) but not (4):

(3) If we are on route 117, then we might be in Maynard.

[3]Gillies (2010) argues instead that $A >$ *must* C is equivalent to $A > C$ itself. See §7 for discussion of this claim.

(4) If we are on route 117, then we might be in Hudson.

(3) says there are some epistemically possible worlds where we are on route 117 and are in Maynard; that is indeed true. (4) says that there are some epistemically possible worlds where we are on route 117 and might be in Hudson; this is false.

As Gillies (2010) observes, examples like (3) and (4) are equivalent to the epistemic possibility of certain conjunctions: $A > \Diamond_E B$ looks equivalent to $\Diamond_E(A \wedge B)$. But there is a small catch: this equivalence likely does not hold if A is not epistemically possible: for then $A > \Diamond_E B$ will be trivially true but $\Diamond_E(A \wedge B)$ will be false. But conditionals presuppose the possibility of their antecedents: (3) and (4) will not be assertable in the first place, if we couldn't be on route 117. Thus I propose we should aim to validate:[4]

Conjunctivitis. $\Diamond_E A \supset ((A > \Diamond_E B) \leftrightarrow \Diamond_E(A \wedge B))$

Analogues of Boxiness and Conjunctivitis seem to hold for historical modals too.

Historical modality is often hard to separate from epistemic modality: after all, if we know A is historically necessary then A will be epistemically necessary; and if A is historically possible, it is rare to know that $\neg A$ is true. But there are ways to prise the two apart. To motivate Boxiness for historical modals, consider the following case.

> *The bet.* You have a coin which is either double-heads or double-tails. You may or may not toss the coin at 12pm. I make two bets. First I bet either that the coin won't land heads: that is, I bet it won't be flipped or it will land tails. I also bet it won't land tails, that it won't be flipped or will land heads. Right now

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[4] A final data point noted by Gillies is that $\Diamond_E$ and $\Box_E$ seem *scopeless* with respect to conditionals. (2) is equivalent to:

(i) It must that if we are on route 62, then we are in Hudson.

(3) is equivalent to:

(ii) Maybe if we are on route 117, we are in Maynard.

Thus, following Gillies (2020), we could attempt to account for the scopelessness of might and must semantically, by validating the following principles:

Scopeless Must. $(A > \Box_E B) \leftrightarrow \Box_E(A > B)$

Scopeless Might. $\Diamond_E A \supset ((A > \Diamond_E B) \leftrightarrow \Diamond_E(A > B))$

The rider that A is epistemically possible is added to Scopeless Might for the same reasons as before.

I am unsure whether to include these in the core restrictor data. It is not obvious the pure restrictor view does validate either of these: either the modal or the if-clause must undergo movement for Functional Application to be possible. Nonetheless they will be theorems of an extension of R. For further discussion, see footnote 16.

it is not settled whether the coin will be flipped (perhaps because that will be settled by the toss of a fair coin.)

I could describe my situation as follows:

(5) Either if the coin is flipped I can't win the first bet; or if the coin is flipped, I can't win the second bet. (I just don't know which.)

Here the modal appears to be restricted; and it does not appear to be epistemic.

The modal is restricted because in no sense is it *unconditionally* true that I will lose at least one bet: if the coin isn't tossed, I win both. Instead (5) says the same thing as:

(6) Either it's not possible for the coin to be flipped and for me to win bet one or it's not possible for the coin to be flipped and for me to win bet two.

This is exactly what Boxiness predicts.

The modal is not epistemic because the coin's being tossed does not *epistemically* necessitate either result. Not knowing what kind of coin it is, I cannot infer a particular outcome simply from its being tossed. But the coin's being tossed does *historically* necessitate the outcome of the bet. For it is settled right now what kind of coin it is. If it is double-headed, it is not historically possible for the coin to be flipped without landing heads; *mutatis mutandis*, the same for the double-tails coin.

To motivate Conjunctivitis, we need to pry apart epistemic and historical possibility. To do so, we can focus on contexts of deliberation. We deliberate between options that are in some sense possible. But not *epistemically* possible: knowing you *won't* do something does not mean it ceases to be an option: simply deciding I will not help you does not automatically remove it as option for me! Options are historical, not epistemic, possibilties.

Now suppose I've been invited to a party tonight, but I decline as I am too tired to enjoy it. By deciding not to go to the party, I come know I will not be there tonight. But that does not mean that it is not *historically* possible that I might go to the party: it is still an option which nothing is preventing me from taking. When we look at natural language judgements, they bear this out:

(7) I could go to the party this evening. I just know that I won't.

(8) I can go to the party this evening. I just know that I won't.

(9) # I might go to the party this evening. I just know that I won't.

Knowing that I won't go removes the epistemic possibility of my going to the party; but it is perfectly consistent with the *historical* possibility of my going.

To make the case for Conjunctivitis for historical modals, now consider:

(10) If I leave work early, I could go to the party. (I just know that I won't.)

(11) If I leave work early, I can go to the party. (I just know that I won't.)

As Conjunctivitis says, (10) and (11) seem equivalent to:

(12) I could leave work early and go to the party. (I just know that I won't.)

(13) I can leave work early and go to the party. (I just know that I won't.)

But (10) and (11) are not epistemic. Compare:

(14) #If I leave work early, I might go to the party. (I just know that I won't.)

2 A Restrictor Logic

I now lay out the logic R, in which we will derive Boxiness and Conjunctivitis as theorems.

R contains standard principles regulating basic conditional inferences. Specifically, R builds on the standard basic conditional logic B, more or less. We have unrestricted versions of the follow axioms and rules from B:[5]

Identity. $A > A$

Agglomeration. If $\vdash (B_1 \wedge ... \wedge B_n) \supset C$ then $\vdash (((A > B_1) \wedge ... \wedge (A > B_n)) \supset (A > C))$

LLE. If $\vdash A \leftrightarrow B$ then $\vdash (A > C) \leftrightarrow (B > C)$

The remaining principles of B are restricted to Booleans:[6]

Simple Cautious Monotonicity. $((A > B) \wedge (A > C)) \supset ((A \wedge B) > C)$, when C is Boolean.

Simple OR. $((A > C) \wedge (B > C)) \supset ((A \vee B) > C)$, when C is Boolean.

Three distinctive axioms form the core of R. First, I add the principle I call Persistence, restricted to Booleans.

Boolean Persistence. $A > (B > A)$, when A is Boolean.

[5] B was first formulated by Burgess (1981); though see also Chellas (1975).
[6] For discussion and motivation of these principles, see Egré and Rott (2021).

Persistence says first supposing A and then B leaves one in a state where A still holds (when A and B are Boolean). This principle is prima facie plausible; consider:[7]

(15) It rained, then if it was cold, then it rained and it was cold.

Given the rest of B, Persistence predicts such sentences to be validities.

Second, I add the following principle, *Flattening*:[8]

Boolean Flattening. $((A \wedge B) > C) \leftrightarrow (A > ((A \wedge B) > C))$, for Boolean A, B and C

Flattening is easier to motivate given LLE. For suppose B entails A: then $A \wedge B$ is equivalent to B. Flattening then says that the result of supposing A and then B is the same as the result of just supposing B, when B entails A. And this indeed seems so:

(16) a. If the coin is tossed, then if it lands heads then I will win the bet.
b. If the coin lands heads I will win the bet.

Finally, I add a principle I call Modal Conditional Excluded Middle:

Modal CEM. $(A > (C > \bot)) \vee (A > \neg(C > \bot))$

In a variably strict theory, this principle says that the closest A-worlds all agree on what worlds are accessible. This principle is derivable from CEM but is strictly weaker. This principle should also acceptable to the restrictor theorist: as we will discuss below, conditionals of the form $\neg A > \bot$ are interdefinable with a kind of necessity operator; and the result of rephrasing this principle to those terms is something predicted by the Kratzer semantics.[9]

Note that all of our restrictions to Booleans are to avoid various triviality results. Boylan (2024) proves that full Persistence leads to triviality, given Identity and Agglomeration. Furthermore, Boylan's results also show the presence of even restricted Persistence forces us to qualify Cautious Monotonicity and OR.[10]

Here then, for ease of reference, is the complete axiomatisation of the R logic:

[7] For further discussion of and motivation for Persistence, see Boylan (2024).

[8] This principle appears in Mandelkern (forthcoming), where it is attributed to Cian Dorr.

[9] The result of rephrasing Modal CEM is the following:

$$(A > \Box C) \vee (A > \Diamond\neg C)$$

According to the Kratzerian, $A > \Box\neg C$ says *at all accessible A worlds, C is true*; so $\neg(A > \Box C)$ says *at some accessible A-worlds, C is false*; and this is the meaning the Kratzerian assigns to $A > \Diamond\neg C$.

[10] Given that Import-Export will also be a theorem, similar restrictions are required to avoid the triviality arguments of Dale (1974), Gibbard (1981), Mandelkern (2018a) and Mandelkern (2021).

Axioms.

PC. All theorems of PC.

Identity. $A > A$

Simple Cautious Monotonicity. $((A > B) \wedge (A > C)) \supset ((A \wedge B) > C)$, when C is Boolean.

Simple OR. $((A > C) \wedge (B > C)) \supset ((A \vee B) > C)$, when C is Boolean.

Boolean Flattening. $((A \wedge B) > C) \leftrightarrow (A > ((A \wedge B) > C))$, for Boolean A, B and C

Simple Persistence. $A > (B > A)$, when A is Boolean.

Modal CEM. $(A > (C > \bot)) \vee (A > \neg(C > \bot))$

Rules.

LLE. If $\vdash A \leftrightarrow B$ then $\vdash (A > C) \leftrightarrow (B > C)$

Agglomeration. If $\vdash (B_1 \wedge ... \wedge B_n) \supset C$ then $\vdash (((A > B_1) \wedge ... \wedge (A > B_n)) \supset (A > C))$

Detachment. $A, A \supset C \vdash C$

A useful theorem of this logic is a restricted version of the Import-Export Principle:[11]

Import-Export. $((A \wedge B) > C) \leftrightarrow (A > (B > C))$, when A, B, C are Boolean.

Instances of this principle, at least for Booleans, are extremely compelling. This version of the principle follows from Persistence, Flattening, Cautious Monotonicity and OR.

To begin to see why this is a restrictor logic, it's helpful note a certain basic restrictor-friendly principle that is a theorem.

It's well-known in conditional logic that we can define a modal operator from the conditional operator.[12] Stipulate that $\Box_> A$ and $\Diamond_> A$ are defined as follows:

Conditional Necessity. $\Box_> A \leftrightarrow_{df} (\neg A > \bot)$

[11] See Appendix A.1 for a proof.

[12] See e.g. Stalnaker (1968) and Lewis (1973). See also Hill (2006), Williamson (2007) and Dorr and Hawthorne (ms.) for attempts to put this fact to philosophical work.

Conditional Possibility. $\Diamond_> A \leftrightarrow_{df} \neg(A > \bot)$

Roughly $\Box_> A$ says that we cannot consistently add the supposition that $\neg A$. This does indeed seem like a kind of necessity: plausibly, supposing $\neg A$ would lead to inconsistency only if A were already necessary. We can also prove that, given the standard axioms contained in R, $\Box_>$ has the logical properties of a necessity operator. $\Box_>$ obeys the K axiom for Booleans and the Necessitation rule unrestrictedly.[13,14]

The distinctive parts of R dictate that this operator behaves in a restrictor-like way.[15] Take the following principle:

Restricted Consequent. $A > \Box_> A$, when A is Boolean.

A restrictor theorist should want such a principle, at least for the basic modals: it says that, in the consequent of a conditional, the conditional necessity operator quantifies only over worlds where the antecedent is true. But Restricted Consequent is easily derived: we have from Persistence that $A > (\neg A > A)$; by Identity and Agglomeration we have $A > (\neg A > \neg A)$; by Agglomeration we have $A > (\neg A > \bot)$, which given our definition of $\Box_>$ just is Restricted Consequent.

3 Deriving the Restrictor Data

Let us now show how to derive Boxiness and Conjuncitivitis.

Both principles are theorems of R, when the modal operators are understood as the conditional modalities:[16]

[13] Proof sketch. First note that for Boolean A and C from OR and Cautious Monotoncity, we can derive the MOD principle:

MOD. $(\neg C > \bot) \supset (A > C)$

K: Suppose $\neg(A \supset C) > \bot$ and $\neg A > \bot$. Then by MOD and Agglomeration $\neg C > \neg(A \supset C) \wedge \neg A$ from which it follows that $\neg C > \bot$.

N: Suppose that $\vdash A$. Then $\vdash \neg A > A$. But by Identity $\vdash \neg A > \neg A$. So by Agglomeration $\vdash \neg A > \bot$.

[14] Adding Strong Centering strengthens the logic to S4 for Booleans. Modus Ponens guarantees the T axiom holds for Booleans. Flattening and Modus Ponens ensures that the 4 axiom holds for Booleans. See Dorr and Mandelkern (2024) for a proof of this claim.

[15] My discussion here draws on that of Boylan (2024), §2.

[16] Furthermore, consider the scopelessness principles for the conditional modalities:

Scopeless $\Box_>$. $\Box_>(A \supset C) \leftrightarrow (A > \Box_> C)$, when A, C are Boolean.

Scopeless $\Diamond_>$. $\Diamond_> A \supset ((A > \Diamond_> C) \leftrightarrow \Diamond_>(A > C))$, when A, C are Boolean.

As shown in Appendix A.1, these are derivable given Simple Strong Centering:

Boxiness$_{\Box_>}$. $(A > \Box_> C) \leftrightarrow \Box_>(A > C)$, when A, C are Boolean.

Conjunctivitis$_{\Diamond_>}$. $\Diamond_> A \supset ((A > \Diamond_> C) \leftrightarrow \Diamond_>(A \wedge C))$, when A, C are Boolean.

Leaving the official derivations to Appendix A.1, I sketch the proofs here. First, for Boxiness, suppose first that $A > \Box_> B$. Given the definition of $\Box_>$, this is equivalent to $A > (\neg B > \bot)$. Applying Import-Export, we derive $(A \wedge \neg B) > \bot$ and then, applying LLE, $\neg(A \supset B) > \bot$. But this is just equivalent to $\Box_>(A \supset B)$. Since this argument is a chain of equivalences, simply run it in reverse to obtain the other direction of Boxiness.

For Conjunctivis, first suppose $A > \Diamond_> B$ and $\Diamond_> A$. By definition of $\Diamond_>$, the former is equivalent to $A > \neg(B > \bot)$. When $\Diamond_> A$, then $A > \neg C$ holds only if $\neg(A > C)$ holds. Thus from $A > \neg(B > \bot)$ and $\Diamond_> A$ it follows that $\neg(A > (B > \bot))$. From Import-Export it then follows that $\neg((A \wedge B) > \bot)$. This is just equivalent to $\Diamond_>(A \wedge B)$. To obtain the other direction, we run the argument in reverse except Modal CEM is used to transition from $\neg(A > (B > \bot))$ to $A > \neg(B > \bot)$.

Our original formulations of these principles were in terms of epistemic and circumstantial necessity, not conditional necessity. But the same arguments can be extended to derive those too. It's generally agreed that the standard reading of the indicative is itself epistemic in flavour. A quick argument from Dorr and Hawthorne (ms.): the following inference seems good.

Must Preservation. $\Box_E C \supset (A > C)$

Consider:

(17) a. The car can't be working properly.
b. So (even) if the car starts, it isn't working properly.

If we assume that the accessibility relation for the standard indicative is epistemic accessibility, it is extremely natural to subscribe to the following principle:

Identity of Accessibility, Epistemic. $\Box_E A \leftrightarrow (\neg A >_E \bot)$, when A is Boolean.

This principle essentially says, for the standard conditional, conditional necessity just is epistemic necessity. Given that, we run the same kind of arguments as above to derive Boxiness and Conjunctivitis for epistemic modals.

Now let's turn to the historical case. We said that the standard reading of the conditional is epistemic; so historical modals are cannot be defined from this conditional. But there is

Boolean Strong Centering. $A \supset ((A > C) \leftrightarrow C)$, when A, C are Boolean.

a very natural way to extend the account. We saw basic modals can take either epistemic or historical accessibility relations. Since conditionals also come with an accessibility relation, it is extremely natural to say the same thing: conditionals can be supplied epistemic or historical accessibility relations, depending on the context. In other words, depending on the context, the conditional might be interpreted with an epistemic accessibility relation, yielding $>_E$, or with a historical accessibility relation, yielding $>_H$. This means that, on the historical interpretation of the conditional, we have the following version of Identity of Accessibility:

Identity of Accessibility, Historical. $\Box_H A \leftrightarrow (\neg A >_H \bot)$, when A is Boolean.

More generally we are then assuming that, for each flavour of accessibility relation, we have a reading of the conditional is coordinated with that flavour: when the conditional interpreted epistemically, conditional necessity is epistemic necessity; when interpreted historicallly, it is historical. This allows us to extend the proofs of Boxiness and Conjunctivitis to historical modals.

There is independent evidence for this historical reading of the conditional. In Moss (2016) and Ciardelli and Ommundsen (2024), it has been noted that bare indicative conditionals give rise to an ambiguity. Consider:

> *Don't Jump!* Jones, the office worker standing on the roof of her 20 story office building, looks down over the edge towards the ground. When we looked half an hour ago, there was no net anywhere near the building; that being said, we cannot be completely certain that there isn't one hanging now. Jones is a known thrill-seeker, but she has no death wish. (Adapted from Moss (2016), §4.3)

It would be very natural in this case to say:

(18) If Jones jumps, she will die.

After all it is a 20 story building and we are pretty sure there is no net! But we can also elicit a contrary reading where in fact it is true to say:

(19) If Jones jumps, she will live.

After all, if Jones *were* to jump that would only be to indulge her thrill-seeking nature, not out of any desire to end her existence. She would only jump if there were a net there to catch her.

As Ciardelli and Ommundsen (2024) note, it is natural to think that the ambiguity is between an epistemic and historical reading of the conditional.[17] (19) seems to be driven by

[17] Note this is not Moss's construal of the case. My diagnosis of the case also diverges in some aspects from that

epistemic considerations: Jones's jumping would provide strong evidence for the presence of a net — why else would she jump? — and so, in turn provides strong evidence that she will survive the jump. Put differently, the vast majority of the *epistemically* possible worlds where Jones jumps are ones where there is a net. Thus (19) is very likely, on an epistemic reading of the conditional.

On the other hand, Jones's jumping in the absence of a net is *historically* possible: even though she has decided against it, jumping is still something that she *can* do. Supposing there is no net, then it is historically impossible that Jones jumps and survive; and it is indeed most likely that there is no net. Thus, most likely it is historically impossible for Jones to jump and survive and so (18) is very likely, when the conditional is read historically.

4 A Restricting Operator Semantics

Now to explore the space of theories validating R. In this section I outline a *restricting operator* semantics, a selection function semantics where conditionals shift both worlds and accessibility relations.[18]

This semantics involves three core technical ideas. First, conditionals are evaluated at pairs of worlds and accessibility relations. Second, the conditional has a variably strict semantics: $A > C$ says that in the "closest" pairs where A true, C is true. Finally, conditionals shift the accessibility relation used to evaluate the consequent. The selected accessibility relation is one that *accepts* the antecedent: it sees only worlds where the antecedent is true, relative to itself. Thus the closest A-world-accessibility relation pair $\langle w, R\rangle$ is one where R accepts A.

Here in more detail is the theory.[19] The conditional has a set selection function semantics, where $A > C$ is true at $\langle w, R\rangle$ just in case the "closest" A-point to $\langle w, R\rangle$ makes C true:[20]

Restricting Operator Semantics. $\langle w, R\rangle \models A > C$ iff for all $\langle w', R'\rangle \in f([\![A]\!], \langle w, R\rangle)$: $\langle w', R'\rangle \models C$

Various constraints are imposed on the selection function to ensure the conditional validates the axioms of R.

of Ciardelli and Ommundsen: unlike them, I do not take this to be an instance of the C/O ambiguity.

[18] This semantics is a generalisation of the semantics from Boylan (2024), which in turn builds on other ways of integrating acceptance into standard conditional semantics, namely Yalcin (2007), Santorio (2022) and Goldstein and Santorio (2021).

[19] In Appendix B.1 soundness for R is proved.

[20] I use bold variables for sets of world-information state pairs. I use $[\![A]\!]$ for the set of pairs expressed by A in a given model.

My first constraint, the Update Constraint, says the selection function shifts the input accessibility relation to one that *accepts* the antecedent, where a point $\langle w, R \rangle$ accepts A iff for all worlds w' R-accessible from w, $\langle w', R \rangle$ makes A true. Define the set of maximal A-accepting subrelations of R:[21]

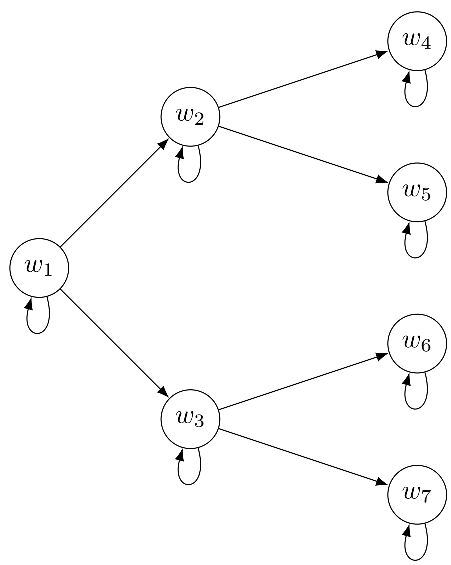
Def. of maximal A-accepting subrelations. $R^{\mathbf{A}} = max\{R' \subseteq R : \text{if } wR'w' \text{ then } \langle w', R' \rangle \in \mathbf{A}\}$

The Update Constraint says that the closest A-point to $\langle w, R \rangle$ is one whose relation parameter is one of these maximal, A-accepting subrelations:

Update Constraint. If $\langle w', R' \rangle \in f(\mathbf{A}, \langle w, R \rangle)$ then $R' \in R^{\mathbf{A}}$.

This guarantees that any point in $f(\llbracket A \rrbracket, \langle w, R \rangle)$ accepts A.[22]

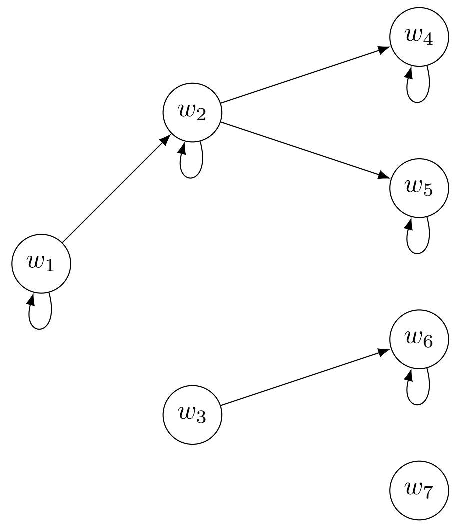
To see how this works with Booleans, take the following frame:



Now let's imagine that p is true at w_1, w_2, w_4, w_5 and w_6. In this case, since p is Boolean, $R^{\llbracket p \rrbracket}$ contains just the following relation:

[21]Note that, when A is not Boolean this is not always a singleton; take for instance $\Box p \vee \Box\neg p$.

[22]$\llbracket A \rrbracket = \{\langle w, R \rangle : \langle w, R \rangle \models A\}$; I suppress reference to a model.



The Update Constraint tells us that, for instance, that the selected p-points at w_1 must have the above relation as their accessibility relation. More generally, when A is Boolean, the effect of updating with A is to remove any arrows going to a world where A is false. This is how the Update Constraint enforces the restricting effect of the conditional: it shifts the relation to one where no worlds where the antecedent is false can be accessed.

The above example shows that the result of updating a relation will tend to leave us with a non-reflexive relation. While every world only sees worlds where the antecedent A is true, this may come at the cost of not seeing itself, if A is false there. My second constraint says that such points can never be selected. In particular, the closest **A**-points are always *proper*:

Propriety. If $\langle w', R' \rangle \in f(\mathbf{A}, \langle w, R \rangle)$ then $w' R' w'$.

These two constraints suffice to derive Identity. For the Update Constraint guarantees that the closest **A**-point always accepts **A**; and at proper points, acceptance guarantees truth. They also suffice for Boolean Persistence. $A > (B > A)$ holds just in case at the closest A points, the closest B points make A true. The Update Constraint tells us that, as we move to the closest A points, we update the accessibility relation with A; we then update it again with B as we move to the closest B points. In the case of Booleans, the update with A is guaranteed to persist through the update with A: thus our final accessibility relation will only see A-points; and given Propriety, this means at the closest A points, the closest B points make A true.

Modal CEM simply requires the selected points to always agree on the accessibility relation:

Relational Uniqueness. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ and $\langle w'', R''\rangle \in f(\mathbf{A}, \langle w, R\rangle)$, then $R' = R''$ and $R'(w') = R'(w'')$.

If there are multiple selected A-points, they must agree on the accessibility relation; and their worlds must see the same worlds via that accessibility relation.

The remaining axioms simply require analogues of the usual variably strict conditions. Mandelkern (forthcoming) shows that, in standard variably strict views, Flattening is characterised by the condition that $f(A \wedge B, w) = f(A \wedge B, w')$, for all $w' \in f(A, w)$. In our framework, something similar suffices for Boolean Flattening. Say that $\mathbf{A}$ is a non-modal proposition when $\langle w, R\rangle \in \mathbf{A}$ iff $\langle w, R'\rangle \in \mathbf{A}$; that is, $\mathbf{A}$ places no constraints on the accessibility relation. Then:

Flat. When $\mathbf{A}$ and $\mathbf{B}$ are non-modal $f(\mathbf{A} \cap \mathbf{B}, \langle w, R\rangle) = f(\mathbf{A} \cap \mathbf{B}, \langle w', R'\rangle)$, for all $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$.

The remaining constraints for Cautious Monotonicity and OR are analogues of their standard variably strict conditions:[23]

CMon. If $f(\mathbf{A}, \langle w, R\rangle) \subseteq \mathbf{B}$, then $f_w(\mathbf{A} \cap \mathbf{B}, \langle w, R\rangle) \subseteq f_w(\mathbf{A}, \langle w, R\rangle)$

Union. $f_w(\mathbf{A} \cup \mathbf{B}, \langle w, R\rangle) \subseteq f_w(\mathbf{A}, \langle w, R\rangle) \cup f_w(\mathbf{B}, \langle w, R\rangle)$

Finally, I assume that we are given a necessity operator $\Box$.

Def. of $\Box$. $\langle w, R\rangle \models \Box A$ iff for all w' such that $wRw' : w', R \models A$.

Think of this operator as analogous to the lexical item "must". In a context where the modal is understood epistemically, the underlying frame is built on an epistemic accessibility relation; mutatis mutandis, when the modal is historical. As we said earlier, we want the interpretation of $\Box$ to coincide with the conditional necessity operator:

Identity of Accessibility. $\Box A \leftrightarrow (\neg A > \bot)$, when A is Boolean.

To validate this principle, I add two more constraints:

Visibility. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' \in R(w)$

[23] As noted, adding Boolean Strong Centering yields the Scopelessness principles. The usual condition for Strong Centering conflicts with the Update Constraint. Appendix B.1 shows the following more complex constraint suffices for Boolean Strong Centering.

Minimality. If $\exists R' \in R^{\mathbf{A}}$ such that $\langle w, R'\rangle \in \mathbf{A}$ then $\exists R' \in R^{\mathbf{A}}$ such that $\langle w, R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ and if $\langle w', R''\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' = w$.

Non-Vacuity. If for some $R' \in R^{\mathbf{A}}$ $R'(w) \neq \varnothing$, then $f(\mathbf{A}, \langle w, R\rangle) \neq \varnothing$

Visibility says that the closest A-points are always ones where the worlds are reachable by the original accessibility relation. Non-Vacuity says that if there are no closest A-points to $\langle w, R\rangle$, then none of the members of $R^{\mathbf{A}}$ relates w to anything. These two together suffice for Identity of Accessibility.

5 How many parameters are shifted?

A natural question now is what, if anything, is at stake in the choice between the semantics above and Kratzer's proposal that conditionals are pure restrictors.

Kratzer is guided by an analogy between conditionals and NP quantifiers. Consider:

(20) Every *adult at the party* was exhasperated.

(21) Some *devious children* fed the birthday cake to the dog.

The italicised material, the *restrictor* of the quantifier, simply serves to restrict the domain of entities the quantifiers range over: (20) ranges over adults at the party in the domain; (21) ranges over devious children in the domain. On Kratzer's theory of the conditional, if-clauses perform the same kind of function but for modal domains: in (1), the modal quantifiers over worlds where that is a dinosaur.[24]

In Kratzer (1981, 2012), conditionals achieve this by shifting a modal base parameter i.e. an accessibility relation. The "if"-clauses simply add a proposition to the accessibility relation. Say that $R + \mathbf{A}$ is the modal base formed by intersecting the outputs of R with $\mathbf{A}$: $R + \mathbf{A}(w) = \{w : \langle w, R\rangle \in \mathbf{A}\}$. Then the restrictor semantics says the following:

Pure Restrictor Semantics. $w, R \models A > C$ iff $w, R + A \models C$

[24]I set aside a further claim sometimes attributed Kratzer, that if-clauses have the *syntax* of restrictors. Recent discussion shows this calim is at best inessential to the pure restrictor view and at worst mistaken.

The syntactic claim is inessential because, as Schulz (2010) notes, the pure restrictor semantics is consistent with a non-restrictor syntax. We can generate Kratzer's syncategoramatic truth-conditions with the following entry:

(i) $[\![if]\!] = \lambda f_{\langle s,stt\rangle}.\lambda p_{\langle\langle s,stt\rangle,\langle\langle s,stt\rangle,st\rangle\rangle}.\lambda q_{\langle\langle s,stt\rangle,\langle\langle s,stt\rangle,st\rangle\rangle}\, q(f + \lambda w.p(f)(g))(g)(w)$

This entry assumes if-clauses do *not* have a restrictor syntax, since the composition assumes the modal is an argument to the if-clause.

Gillies (2010) notes not all restricted readings are consistent with the syntactic hypothesis. Consider:

(ii) If that is a dinosaur, it can't be a brontosaurus but it might be t-rex.

What exactly is the syntactic analysis here, if the if-clause is the restrictor for a modal?

This nicely captures the data we mentioned above. In fact, with a few caveats, it gives more or less the same axiomatic explanation of why the restrictor data hold.[25]

The main technical difference between this view and mine lies in how many parameters are shifted. For Kratzer conditionals are *pure* restrictors because if-clauses shift conditional domains and nothing else. On the restricting operator semantics, the world parameter is also shifted.

These views make very different predictions about bare conditionals. As Kratzer herself noticed, if conditionals are pure restrictors, conditionals without any modals should simply be equivalent to their consequents. Consider a bare conditional like:

(22) If it rained, then they all got wet.

Prima facie, there is just no modal to restrict here, with the result the if-clause is vacuous. Kratzer's famous fix is to posit a silent modal here. The real syntactic form of (22) is:

[25] Let me start by outlining what seems to me the logic of the pure restrictor view, which I'll call R_P.

First, on the pure restrictor view, $\Box_>$ is a primitive operator, not one derivable from the conditional. This is because on the pure restrictor view $>$ has no modal force by itself. We stipulate that this operator obeys K at least for Booleans.

Second, we expand the axioms of R, adding:

Boxy Identity. $A > \Box_> A$ for Boolean A.

Boxy CM. $((A > \Box_> B) \wedge (A > \Box_> C)) \supset ((A \wedge B) > \Box_> C)$, when A, B, C are Boolean.

Boxy OR. $((A > \Box_> C) \wedge (B > \Box_> C)) \supset ((A \vee B) > \Box_> C)$, when A, B and C are Boolean.

Boxy Flattening. $((A \wedge B) > \Box_> C) \leftrightarrow (A > ((A \wedge B) > \Box_> C))$ when A, B and C are Boolean.

Boxy Persistence. $A > (B > \Box_> A)$, when A is Boolean.

Boxy Modal CEM. $(A > (C > \Box_> \bot)) \vee (A > \neg(C > \Box_> \bot))$

Note that each is the result of taking an old axiom and, for Boolean A and C, replacing any conditional of the form $A > C$ with $A > \Box_> C$. Finally we also add the axiom:

Box Swap. $\Box_> A \leftrightarrow (\neg A > \Box_> \bot)$

As the reader can confirm for themselves, in this system we can derive the following form of Import-Export:

Boxy Import-Export. $(A > (B > \Box_> C)) \leftrightarrow ((A \wedge B) > \Box_> C)$

This means that we can run almost the same proofs as we did in R to derive Boxiness and Conjunctivitis. (In the proof of Conjunctivitis, it may help to note that $A > \bot$ entails $A > \Box_> \bot$ in this system.)

In fact, R is equivalent to R_p, given Identity of Accessibility. For all of the extra axioms of R_p can be derived in this setting: Boxy Identity, CM, OR, Flattening, Persistence and Box Swap are easily derived given Boolean Import Export and Identity of Accessibility. Boxy Modal CEM is then easily derived given Box Swap and Identity of Accessibility.

(23) If it rained, then MODAL they all got wet.

In her classic papers, Kratzer takes it to be something like an epistemic *must*. *Some* modal must be present, anyway, for the pure restrictor view to give plausible truth-conditions to apparently bare conditionals.

Notice, though, that no modal is required on the restricting operator view. When a conditional contains no modal, the shift in accessibility relation is redundant, but the shift in world is not. On the restricting operator, bare conditionals have the truth-conditions of standard variably strict views: (22) will be true iff in the closest world where it rained, everyone got wet. Thus clearly says something different from its consequent.

This is a distinction with a significant difference. Even though many linguists accept Kratzer's silent modals, it seems to me that bare conditionals still pose a fairly serious challenge to the pure restrictor view.

Bare conditionals should be one of the central test cases for the pure restrictor hypothesis. To determine whether *all* that conditionals do is restrict modal domains, surely a decisive test is whether if-clauses contribute anything when no modals are present. Imagine we observed that bare conditionals were unintelligible, that in fact all conditional sentences always come with an (overt) modal in the consequent: would this not be the best possible evidence we could get for the restrictor view? The flip side of this must be that if bare conditionals are intelligible, then we have evidence *against* the pure restrictor view. We cannot have it both ways.

Now, as the philosophy of science teaches us, no hypothesis is testable in isolation: we can always reject some auxiliary hypothesis. Indeed, in positing a silent modal, this is effectively what Kratzer does. But we should be wary of trying shielding our theories from otherwise decisive test cases this way: absent some strong independent motivation, this runs the risk of making our hypotheses untestable.

To my knowledge, we have no independent motivation for Kratzer's covert modals. If anything, this auxiliary hypothesis is independently shaky. A variety of authors, including Schulz (2010) and Ciardelli (2021b), note that, without heavily restricting the places where this modal can appear, the theory massively overgenerates modal readings of non-conditionals. For instance, take a disjunction like:

(24) Either it's raining or it's snowing.

It's not obvious that this has a reading where it means:

(25) Either it must be raining or it must be snowing

Or take:

(26) I doubt that it's raining.

This doesn't seem to have a reading where it says:

(27) I doubt that it must be raining.

There are of course things that can be said. Perhaps postulating the modal is a line of last defence, something that only happens to prevent redundancy. Or perhaps the silent modal is not *must* but rather a covert form of Cariani and Santorio (2018)'s selectional modal, whose presence would only make a truth-conditional difference in conditional consequents.[26] But these are all defensive maneouvres: we were looking for some *independent* evidence that apparently bare conditionals are not truly bare. So far none is forthcoming.

Were the pure restrictor view our only option, perhaps we could stomach its problems: it does give a strong and simple explanation of conditional restriction. But we have another option now that better fits the contours of the data. We saw that conditionals restrict modals: this is good prima facie evidence that conditionals restrict something like an accessibility relation. But we also saw that conditionals are not redundant when there is no modal to shift: this suggests that modal restriction is not the *only* job of conditionals, just as the restricting operator view says.[27]

[26] See Mandelkern (2016) and Cariani (2021) for versions of this view.

[27] Alternatively, to maintain that conditionals are pure restrictors, we might try some other way to deriving the modal force. There are two important attempts to do this: the view of Ciardelli (2021c,d); and the view of McGee (1985), as developed by Cantwell (2008). Both are extremely intriguing, deserving of more extensive comparison; but I will briefly report some worries about both.

Ciardelli says modal force is a shiftable parameter of the index. Roughly, an atomic p is true relative to an information state s and a modal force of possibility ($\exists$)/necessity ($\forall$)/probability (π) iff p is consistent with/entailed by/made likely by s. Modals simply shift this quantificational force: $\Diamond p$ is true relative to s and f just in case p is true relative to s and $\exists$; $\Box p$ is true relative to s and f just in case s is true relative to s and $\forall$. Conditionals simply add their antecedent the information parameter s.

My first issue is that theory is not developed to handle restriction of non-epistemic modalities, like historical modality and q-adverbs; and I myself am not sure how that would go. My second is a more technical concern about the shifting modal force parameter. Ciardelli observes that when we have multiple modals the outermost modal will be redundant: the innermost modal will undo the shift of the outer modal. Ciardelli is correct that iterated constructions like $\Diamond\Box A$ are not especially natural and so do not pose a clear problem. However, a more serious problem arises if we interpose a conditional between the two modals: $\Diamond(A > \Box C)$ is rendered equivalent to $A > \Box C$; $\Box(A > \Diamond C)$ is rendered equivalent to $(A > \Diamond C)$. Neither prediction seems correct to me.

Cantwell, following McGee, recovers the modal content by hardwiring it into the semantics of *atomics* in an extremely subtle way. Conditionals simply add their antecedents to a premise set. An atomic sentence p is true relative to w and a premise set Γ just p is true at the closest world to w where all sentences in Γ are true. But relative to the default setting for the premise set, the empty set of premises, the selected world is simply the world of evaluation. Thus unembedded atomics exhibit no modal force; their modal force only emerges in the consequents of conditionals. (So an atomic p just has the truth-conditions of "will p" on Cariani and Santorio (2018)'s selection function semantics for "will".)

My main complaint about this style of view is the logic it yields: as far as I can tell, any version of this view will have to give up both Identity and Persistence. But I leave this only as a tentative argument against.

This, then, is the score so far. The restricting operator semantics predicts the restrictor data for both epistemic and historical conditionals; and it does so without any appeal to covert modals. Instead it appeals to independently attested epistemic and historical readings of indicatives. But the game is of course not over. In this paper, I will consider one further important case of restriction: deontic modals.

6 Deontic Conditionals

Deontic conditionals, like conditionals with necessity consequents, are also felt in some sense to be *non*-conditional. Consider:[28]

(28) a. I should visit my grandmother.
b. But if I don't, I should at least call her to say I am not coming.

In (28-b), I'm not saying that from my not going it would *follow* that I should tell her I'm not going.[29] Rather I just seem to be making a different sort of *ought* claim: amongst a certain set of worlds, the best worlds are those where I call to say I'm not going.

This section will mirror what came before. I start with an axiomatic principle stating the intuitive truth-conditions of deontic conditionals; I give a logic, R_O, which allows us to derive that principle; and we will see how the restricting operator view can validate R_O.

6.1 Regimentation

With deontic conditionals, the intuitive truth-conditions are more difficult to regiment. It's not obvious natural language alone offers us the tools for the job. Consider first a deontic form of Boxiness:

Deontic Boxiness. $A > \mathcal{O}C \leftrightarrow \mathcal{O}(A \supset C)$

Deontic Boxiness leads to absurd consequences, given very weak principles of deontic logic. Suppose we allow that logical equivalents are substitutable under $\mathcal{O}$: we then get the following.

Monotonicity. $\mathcal{O}A \supset (\neg A > \mathcal{O}A)$

For suppose I ought to visit my grandmother, $\mathcal{O}V$. V is equivalent to the material conditional $\neg V \supset V$. So, by substitution of logical equivalents, it follows that $\mathcal{O}(\neg V \supset V)$. But

[28] Inspired of course by Chisholm (1963).
[29] See in particular Willer (2014) for nuanced further discussion here.

now from Deontic Boxiness it follows that if I don't visit my grandmother, I ought to visit my grandmother. This is wrong: if I don't visit grandmother, I ought to do something *else*. Deontic conditionals are non-monotonic.

Boxiness seems to get something right: for any given A, $A > \mathcal{O}C$ feels equivalent to some deontic necessity claim $\mathcal{O}'(A \supset C)$. The problem is that the necessity operator $\mathcal{O}'$ is not the same for each antecedent A. In (28-b), the deontic conditional is equivalent to a claim about what you ought to do, relative to a body of information that holds fixed that you will not visit your grandmother. This of course is not simply what is entailed by your current obligations plus the claim that you are not coming.

I suggest we simply stipulatively introduce a set of deontic operators $\mathcal{O}^A$, where each operator is indexed to a given proposition A. Intuitively $\mathcal{O}^A C$ says that, given our background information plus the proposition that A, C ought to be the case: it is the proposition expressed by "ought C" in a context with the added information that A. Thus $\mathcal{O}$ is a dyadic deontic operator, of the kind studied by, among many, many others, von Wright (1951), Chisholm (1963), Hansson (1969), Fraassen (1972) and Lewis (1974).[30]

Our target principle is stated in terms of the dyadic *ought*:

Iffy Ought Thesis. $A > \mathcal{O}C \leftrightarrow \mathcal{O}^A C$, where A and C are Boolean.

This principle says that conditionals with unary deontic consequents are equivalent to dyadic deontics. This captures our judgement that in (28-b) the conditional expresses a kind of necessity. Given the Iffy Ought Thesis, the deontic conditional is equivalent to the claim that it is deontically necessary that I call, relative to the information that I do not go.

6.2 Derivation

I now show how to extend R logic to derive the Iffy Ought Thesis.

We first need some assumptions governing the dyadic *ought*.

O-Normality. $\mathcal{O}^A \top$

O-MOD. $\mathcal{O}^{\neg C} \top \supset \mathcal{O}^A C$, where A and C are Boolean.

O-CSO. $((\mathcal{O}^A B \wedge \mathcal{O}^B A) \wedge \mathcal{O}^A C) \supset \mathcal{O}^B C$, where A, B and C are Boolean.

O-Normality says that, given any body of information, the tautology ought to be the case. MOD says that if what ought to be is inconsistent given our background information plus $\neg C$ then C ought to be the case given any body of information. O-CSO says that if B ought

[30]Note that, unlike the conditional deontic logic tradition, we do not *define* this dyadic operator in terms of the conditional.

to be the case given A and vice versa, then the same things ought to be true given either A or B. As is standard, I assume the unary ought is definable from the dyadic: $\mathcal{O}A$ holds iff $\mathcal{O}^{\top}A$ holds.

We also need to connect deontics with the basic notions of necessity. I assume *ought*s come in both epistemic and historical flavours. Specifically, the accessibility worlds for $\mathcal{O}$ are a subset either of what is epistemically or of what is historically accessible. Given our Identity of Accessibility principles, we can capture this assumption with the following principle, using the subscript F to mark an arbitrary flavour:

Must entails Ought. $(\neg A >_F \bot) \supset \mathcal{O}_F A$

In words: if it is epistemically/historically necessary that A then it ought epistemically/historically to be that A.

Given R plus the above principles, the Iffy Ought Thesis is equivalent to a principle I call No Self-Defeat.

No Self-Defeat. $\mathcal{O}^A C \leftrightarrow (A > \mathcal{O}^A C)$, where A and C are Boolean.

In variably strict terms, No Self-Defeat says that what actually ought to be the case relative to A is the same as what ought to be the case relative to A in the closest A-worlds.

To see that No Self-Defeat is necessary and sufficient for the Iffy Ought Thesis, first note that the following principle is a theorem of R plus O-Normality, O-MOD, O-CSO and Must entails Ought:

Interchange. $(A > \mathcal{O}^A C) \leftrightarrow (A > \mathcal{O}C)$

Interchange says if A, what unconditionally ought to be is what ought to be conditional on A. Interchange follows given Must entails Ought and the fact that $\mathcal{O}^A C$ has the basics of a conditional logic. But given Interchange, it is clear that No Self-Defeat is necessary and sufficient for the Iffy Ought Thesis.

I propose that, extended to deontics, the logic of the restrictor view is the result of adding to R O-Normality, O-Mod, O-CSO, Must entails Ought and No Self-Defeat; I call this logic R_O. As we have seen, the Iffy Ought Thesis is a theorem of R_O. There is also a striking similarity between this logic and R itself. The restrictor behaviour of the basic modals is driven by Persistence and Flattening. R_O gives us a deontic analogue of Persistence. Given Must entails Ought, it's straightforward to show that we have as a theorem:

$$A >_F \mathcal{O}^C_F A$$

To make the analogy even more obvious, we could even rewrite $\mathcal{O}^C$ as $C \Rightarrow$:

$$A >_F (C \Rightarrow_F A)$$

No Self-Defeat is structurally similar to Flattening. Rewrite No Self-Defeat in terms of ⇒:

No Self-Defeat$_{\Rightarrow}$. $A \Rightarrow_F C \leftrightarrow A >_F (A \Rightarrow_F C)$

No Self-Defeat is a multi-modal analogue of a special case of Flattening.

6.3 Adding No Self-Defeat

Let's now examine how the restricting operator theorist might validate Self-Defeat.

On a very natural theory of the dyadic ought, $\mathcal{O}^A C$ says the best accessible A-worlds are C-worlds. More carefully, define the best accessible worlds, relative to a pair $\langle w, R\rangle$, as follows:

Def. of $\mathcal{B}$. $\mathcal{B}(w,R) = \{w' \in R(w) : \neg\exists w'' \in R(w) : w'' <_{w,R} w'\}$

Then let $\mathcal{O}^A C$ say that the best accessible worlds, relative to the accessibility relation updated with A, are C worlds:[31]

Semantics for $\mathcal{O}^A C$. $w, R \models \mathcal{O}^A C$ iff for all $R' \in R^{[\![A]\!]}$ for all $w' \in \mathcal{B}(w, R')$ $w', R' \models C$

Since we are extending R, we keep all the constraints on selection functions from §4. Moreover, note that Must entails Ought, O-Normality and O-MOD are trivially valid, given the above semantics for $\mathcal{O}$. To ensure O-CSO is valid for Booleans, we impose the following constraint:[32]

α. If $\mathbf{A} \subseteq \mathbf{B}$ and if $\mathcal{B}(w, R^{\mathbf{B}}) \subseteq \downarrow[\![A]\!]$ then $\mathcal{B}(w, R^{\mathbf{A}}) = \mathcal{B}(w, R^{\mathbf{B}})$.

How might we validate No Self-Defeat? First of all consider the following principle. Let $\downarrow\mathbf{A}$ be the set of worlds contained in some pair in $\mathbf{A}$; then:

Tracking. If $w' \in f_w(\mathbf{A}, \langle w, R\rangle)$ then $R(w) \cap \downarrow\mathbf{A} = R(w') \cap \downarrow\mathbf{A}$, where $\mathbf{A}$ is a non-modal proposition.

[31]Note that, unlike on Kratzer's semantics, the ordering is a function of the accessibility relation; thus shifts in the accessibility relation can induce shifts in the ordering. I take the extensive literature on the miners problem, including Kolodny and MacFarlane (2010), Cariani et al. (2013) and Cariani (2016), to have established this as desirable.

[32]From Sen (1971).

Roughly, Tracking says that moving to the closest A-worlds can't change what A-worlds are accessible (when A is Boolean). Appendix B.2 shows Tracking is a necessary condition for No Self-Defeat. It is also shows that Tracking becomes sufficient for No Self-Defeat given a principle on ordering relations:

Supervenience. If $R(w) = R'(w')$ then $w_1 \leq_{w,R} w_2$ iff $w_1 \leq_{w',R'} w_2$

Supervenience says that the ordering supervenes on information-states: no difference in an ordering, relative to a world-accessibility relation pair, without a difference in the information at that pair. Thus, adding Tracking and Supervenience to our existing constraints suffices to derive No Self-Defeat.

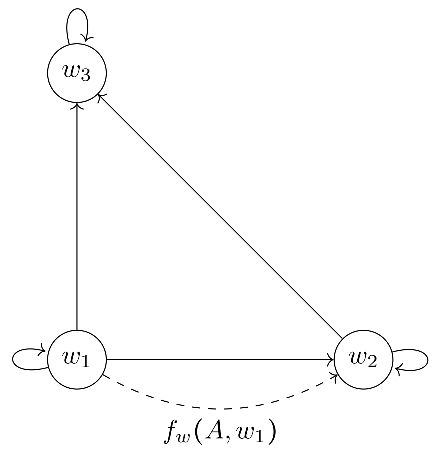
Both principles are entailed by the standard theory of historical modals. Tracking holds trivially for any R that is an equivalence relation. And historical accessibility is an equivalence relation: w' is historically accessible from w at some point t just in case w' and w are historical duplicates up until t; and historical duplication is clearly an equivalence relation.

Supervenience holds because the sources of deontic value at t are settled by the facts at t: for example, whether it would be good or permissible for Alice to skip school is fixed by facts about the *present*, such as what the school rules are, what her parents have said to her and so on. Any other world where all the same worlds are historically accessible will be a duplicate of this world; and so the morally facts in that world, that is, the rules at t, her parents strictures up to t and so on, will be the same.

In the epistemic case, both principles have more controversial upshots but both I take to be defensible. Here I sketch the motivation in favour of each principle.

Start with Tracking, which as we saw says that moving to the closest A-worlds does not change what A-worlds are accessible. Do not confuse this with a more controversial claim. Tracking does not say that the degree of support one's evidence gives to A cannot change on moving to the closest A-worlds.

Consider the diagram below, where I omit only transitive arrows.



Appendix B.2 fills this basic structure into a frame where Tracking holds. Now, letting $A = \{w_2, w_3\}$, notice that while w_1 and w_3 see the same A-worlds, they do not see the same $\neg A$ worlds. This happens here because R is not symmetric: w_1 sees w_3 but not vice versa. This is consistent with Tracking, which tells us only that the accessible A-worlds remain the same at the closest A-points.

The evidential support of A at w_1 will likely differ from that at w_2. For concreteness, take an urprior Pr that assigns equal weight to all three worlds; and say the evidential probability of a proposition B at w is $Pr(B|E(w))$.[33] The evidential probability of A at w_1 is 0.6. At w_3 the evidential probability is 1: since w_3 does not see w_1, no $\neg A$ worlds remain. Thus the probability of A has increased by moving to the closest A-world.[34]

It is welcome that Tracking does not require symmetry: symmetry is widely rejected for epistemic accessibility.[35] However Tracking does place some controversial constraints on accessibility. First, R must be transitive. While transitivity is controversial in epistemic logic, most likely we will have to stomach its addition anyway: the 4 axiom is derivable if we have both Boolean Modus Ponens and Flattening. More interestingly, Tracking requires R to be connected.[36] This principle fails on the margin for error frames of Williamson (2000, 2013); but it holds on the model of knowledge given in Stalnaker (2006, 2015). Thus, while Tracking will not be acceptable to all, I do take it to be defensible. Its plausibility should certainly increase, once it is distinguished from the much more controversial claim above.

Supervenience is in fact defensible for epistemic oughts, once we have a clear view of

[33] This is the approach of Williamson (2000, 2011).

[34] But support for A cannot *decrease* as we move to the closest A-worlds. To me, this seems relatively intuitive. If it seems otherwise to you, notice that this is inevitable whenever we have transitivity: for the probability of A to decrease, w_3 would have to access new $\neg A$-worlds. But this would be a violation of transitivity.

[35] See for instance McDowell (1983), Williamson (2000) and Weatherson (ms.).

[36] R is connected iff when w_1Rw_2 and w_1Rw_3, then either w_2Rw_3 or w_3Rw_2.

what an epistemic ought is. Epistemic deontics are subjective oughts and thus depend on two things: my evidence and some procedure that tells me in how to make prudent or moral decisions given my evidence. If I epistemically ought to buy home insurance, then in light of my evidence and my sensitivity to risk, buying insurance is the most sensible thing to do; no matter if, unbeknownst to me, my home will suffer no catastrophes in my lifetime. If I epistemically ought to block neither shaft in the miners case, then, even if they are in fact in shaft A, my evidence and my decision rule direct me to block neither shaft.

But in fact, these two things may simply be one. My evidence by itself knows enough about my decision rule to tell me what to do: I know enough about my risk tolerance to know I do not want to forego the insurance; and I know roughly how much risk I feel is morally permissible to subject the miners to. Any situation where I have the same evidence is one where I know the same things about my decision rule; thus what I epistemically ought to do will be the same.

The controversial cases are those where I do *not* know what the correct decision rule tells me to do, cases of *normative uncertainty*. Supervenience holds here if what I should do is determined by what what is best in light of my evidence; and I am sympathetic to the arguments of Pittard and Worsnip (2017) that this is indeed so.[37] But such cases are not the bread and butter of semantics: it is not obviously the job of semantics to align with what the correct theory says in these cases.

7 Comparison to Gillies 2010

Let us consider an important precedent to my discussion, Gillies (2010). Gillies shows how a kind of restricing operator semantics captures similar restrictor data. But there are some important differences between what he and I aim to do; and there are important differences in the kind of restricting operator semantics we offer.

I provided an axiomatisation of the restrictor view. This helps us better understand, from a logical point of view, what features of a semantics are really doing the work in capturing the restrictor data. Gillies's discussion, while illuminating, provides no axiomatisation: it focuses solely on providing a semantics that predicts various restrictor-like data.

The axiomatisation clarifies the issues here, as we can see from comparing Gillies' discussion to mine. At various points, Gillies seems to suggest that an important component of a restrictor semantics is a strict semantics, where $A > C$ says, roughly, that $\Box_E(A \supset C)$. My axiomatisation helps us see that this is not the case. A variably strict semantics will do the trick, if it validates R.

[37] See also Sepielli (2009, 2013); Weatherson (2019) is a dissenting voice here.

Second, Gillies does not aim to provide an account of how non-epistemic modals are restricted. Indeed, as Khoo (2011) notes, it is not entirely obvious how to extract an account of deontics from Gillies: various assumptions he makes, such as the assumption that modal bases are closed, are not plausible in the deontic case. My account is readily extendable to deontics, as we have seen.[38]

A final, but related, difference in aims lies in the particular principles we aim to validate. Gillies validates:

If/Must. $(A > C) \leftrightarrow (A > \Box_E C)$

A natural principle for a strict theorist to want, but one that is independent of the restrictor data. As we have seen, Boxiness suffices to explain restrictor data for basic necessity modals. And Boxiness is not equivalent to If/Must: my theory validates Boxiness but not If/Must: $A > C$ says that all the *closest* A-worlds are C-worlds; it does not say that all *accessible* worlds are C-worlds.

Restrictor theorists should welcome this, as Rothschild (2015) observe If/Must is suspect. Compare:

(29) a. If the coin is flipped, it will land heads.
b. If the coin is flipped, it must land heads.

The latter is clearly stronger than the first: if the coin landed heads, I would win a bet on the first, but not on the second.

In terms of the systems we provide, mine is more general in various places. Gillies' system is strict, but mine is variably strict. A strict system can be recovered by adding constraints to the selection function. A variably strict semantics cannot be obtained by relaxing constraints on the strict semantics. Gillies' system assumes an S5 logic for the epistemic modal; mine is consistent with a weaker logic for the various modalities. (A good thing too, given the controversies surrounding S5.) These differences again help clarify what assumptions are really required by the restrictor view.[39]

[38] A question left outstanding in this paper is how the view might be extended to q-adverbs. In XXX, I argue that the view in this paper can capture q-adverbs by postulating, in addition to epistemic and historical readings of the conditional, a q-adverbial reading.

[39] A final difference is that, for Gillies, updating with the antecedent is not update to *acceptance*; Mandelkern (2021, forthcoming) shows that as a result Identity fails in certain cases. My view validates Identity.

8 The C/O Ambiguity

One last order of business is to rebut an argument for covert modals. Geurts (2004) and others note that certain q-adverbial and deontic conditionals *lack* restricted readings. Many take this as an argument for the pure restrictor view's covert modals. I show the restricting operator view can derive these readings too: these are simply cases where the flavour of the embedded modal *differs* from the flavour of the conditional.

Zvolenszky (2002) sketches the following kind of case: Billie Eilish's advertising contract with Pepsi specifies that she only drink Pepsi in public. Here (30) is true and (31) is false:

(30) (Even) if Billie Eilish drinks Coke in public, she should drink Pepsi in public.

(31) If Billie Eilish drinks Coke in public, she should drink Coke in public.

As Geurts (2004) notes, the Kratzerian can see this as evidence for a covert modal that is being restricted:(30) and (31) contain an extra silent epistemic modal, embedding the entire overt consequent. This is why the deontics are unrestricted: some other modal absorbs the restricting force of the if-clause.[40]

Following Geurts, let us call these readings of (30) and (31) *C-readings*. Are C-readings not evidence for the covert modals story?

They are not: the restricting operator semantics can account for them too. In deriving our restrictor principles, a crucial assumption was that the flavour of the conditional matched the flavour of the modal. For example, to derive the epistemic/historical Iffy Ought principle, we assumed we had an epistemic/historical deontic inside an epistemic/historical conditional. While a natural default, the modal and conditional flavours do not *have to* match; and when they don't, the C-readings emerge.

Return to (30) and (31). Given a historical deontic, the following interpretations are available:

(32) $Coke >_E \mathcal{O}_H(Pepsi)$

(33) $Coke >_E \mathcal{O}_H(Coke)$

Here the deontics are unrestricted. The conditional restricts the epistemically possible worlds, but doesn't thereby restrict the historically possible worlds, the predomain of $\mathcal{O}$. Thus (32)

[40] Q-adverbs also give rise to this ambiguity: sentences like the following have readings.

(i) If Beryl is in France, she often visits the British Museum.

As noted in footnote 38, I aim to capture q-adverbial conditionals by positing adverbial readings of the conditional connective. This allows me to straightforwardly capture C/O readings of adverbial conditionals: the "C" reading is simply a case where the flavour of the conditional is not q-adverbial.

says in the closest epistemically possible worlds where Billie drinks coke, she should drink Pepsi given what is (unrestrictedly) historically possible. Regardless of what she does indeed drink, the actual facts say she should drink coke and so this claim is true. Similarly, (33) says that in the closet epistemically possible worlds where she drinks Coke, she should drink Coke; this we know to be false.

The so-called C/O ambiguity is really an ambiguity between structures where the modal is restricted or unrestricted. To generate these unrestricted readings in a Kratzer framework, an intermediate covert modal is posited to absorb the force of the conditional.[41] In the restricting operator framework, no covert modals are needed: simply allow the modal and the conditional differ in modal flavour.

[41] Here I have been helped by remarks in Cariani (2025).

A Axiomatic proofs

A.1 §2 Results

Fact 1. $R \vdash$ Simple Import Export.
Proof. First, note that the following is derivable in R, using Simple Cautious Monotonicity and OR:

Simple Cautious Transitivity. $((A > B) \wedge ((A \wedge B) > C)) \supset (A > C)$, when C is Boolean.

Now assume A, B and C are Boolean. Then:

1.	$A > (B > C)$	assumption
2.	$A > (B > A)$	Persistence
3.	$A > ((A \wedge B) > C)$	S Cautious Mon, Aggl, LLE, 1,2
4.	$(A \wedge B) > C$	B Flattening, 3
5.	$A > (B > C) \supset (A \wedge B) > C$	Conditional Proof: 1-5
6.	$(A \wedge B) > C$	assumption
7.	$A > ((A \wedge B) > C)$	B Flattening, 6
8.	$A > (B > C)$	S Cautious Trans, Aggl, LLE, 2,7
9.	$(A \wedge B) > C \supset A > (B > C)$	Conditional Proof: 6-7
10.	$A > (B > C) \leftrightarrow (A \wedge B) > C$	PL, 5,9

Recall:

Boxiness$_{\Box_>}$. $A > \Box_> C \leftrightarrow \Box_>(A \supset C)$, where A and C are Boolean.

Conjunctivitis$_{\Diamond_>}$. $\Diamond_> A \supset ((A > \Diamond_> C) \leftrightarrow \Diamond_>(A \wedge C))$ where A and C are Boolean.

Fact 2. $R \vdash$ Boxiness$_{\Box_>}$
Proof. Assume A and C are Boolean. Then:

1.	$\Box_>(A \supset C)$	assumption
2.	$\neg(A \supset C) > \bot$	Identity of Accessibility
3.	$(A \wedge \neg C) > \bot$	LLE

4. $A > (\neg C > \bot)$ Boolean Import Export

5. $A > \Box_{>} C$ Identity of Accessibility

6. $\Box_{>}(A \supset C) \supset (A > \Box_{>} C)$ conditional proof, 1-5

7. $A > \Box_{>} C$ assumption

8. $A > (\neg C > \bot)$ Identity of Accessibility

9. $(A \wedge \neg C) > \bot$ Import-Export

10. $\neg(A \supset C) > \bot$ LLE

11. $\Box_{>}(A \supset C)$ Identity of Accessibility

12. $(A > \Box_{>} C) \supset \Box_{>}(A \supset C)$ conditional proof, 7-11

13. $(A > \Box_{>} C) \leftrightarrow \Box_{>}(A > C)$ PL, 6,12

Fact 3. $R \vdash$Conjunctivitis$_{\Diamond_{>}}$
Proof. Assume A and C are Boolean. Then:

1. $\Diamond_{>} A$ assumption

2. $A > \Diamond_{>} C$ assumption

3. $\neg(A > \bot)$ Identity of Accessibility, 1

4. $A > \neg(C > \bot)$ Identity of Accessibility, 2

5. $\neg(A > (C > \bot))$ 3,4, Agglomeration, PL

6. $\neg((A \wedge C) > \bot)$ Boolean Import Export

7. $\Diamond_{>}(A \wedge C)$ Identity of Accessibility

8. $\Diamond_{>} A \supset \Diamond_{>}(A \wedge C)$ conditional proof, 2-7

9. $\Diamond_{>}(A \wedge C)$ assumption

10. $\neg((A \wedge C) > \bot)$ Identity of Accessibility, 9

11. $\neg(A > (C > \bot))$ Boolean Import Export, 10

12. $A > \neg(C > \bot)$ Modal Excluded Middle, 11

13. $A > \Diamond_> C$ Identity of Accessibility, 12

14. $\Diamond_>(A \wedge C) \supset A > \Diamond_> C$ conditional proof, 9-13

15. $((A > \Diamond_> C) \leftrightarrow \Diamond_>(A \wedge C))$ 8,14

16. $\Diamond_> A \supset ((A > \Diamond_> C) \leftrightarrow \Diamond_>(A \wedge C))$ PL, 1-15

Recall also:

Scopeless $\Box_>$. $A > \Box_> C \leftrightarrow \Box_>(A > C)$

Scopeless $\Diamond_>$. $\Diamond_> A \supset ((A > \Diamond_> C) \leftrightarrow \Diamond_>(A > C))$

Fact 4. R + Boolean Strong Centering $\vdash$ Scopeless $\Diamond_>$

Proof. Note first that Boolean SC suffices to derive Boolean Modus Ponens. Recall also, as noted in footnote 14, that in R+ Boolean SC $\Box_>$ has an S4 logic for Booleans. Assume A and C are Boolean. Then:

1. $\Box_>(A > C)$ assumption
2. $\Box_>((A > C) \supset (A \supset C))$ Modus Ponens, Necessitation
3. $\Box_>(A \supset C)$ K, 1,2
4. $A > \Box_> C$ Boxiness, 3
5. $\Box_>(A > C) \supset (A > \Box_> C)$ conditional proof, 1-4
6. $A > \Box_> C$ assumption
7. $\Box_>(A \supset C)$ Boxiness, 6
8. $\Box_> \Box_> (A \supset C)$ 4 axiom, 7
9. $\Box_>((A > \Box_> C) \leftrightarrow \Box_>(A \supset C))$ Boxiness, Necessitation
10. $\Box_>(A > \Box_> C)$ K, 8,9
11. $\Box_>(A > C)$ K, T, Agglomeration, 10
12. $(A > \Box_> C) \supset \Box_>(A > C)$ conditional proof, 6-11
13. $A > \Box_> C \leftrightarrow \Box_>(A > C)$ PL, 5,12

Fact 5. R + Strong Centering $\vdash$ Scopeless $\Box_>$

Proof. Assume A and C are Boolean. Then:

1. $\Diamond_> A$ assumption
2. $A > \Diamond_> C$ assumption
3. $\Diamond_>(A \wedge C)$ Conjunctivitis, 1,2
4. $\Diamond_>(A > C)$ Strong Centering, 3
5. $A > \Diamond_> C \supset \Diamond_>(A > C)$ conditional proof, 2-4
6. $\Diamond_>(A > C)$ assumption
7. $\Diamond_>(A > \Diamond_> C)$ T axiom, Agglomeration
8. $\Diamond_> \Diamond_> A$ 4 axiom, 1
9. $\Box_>(\Diamond_> A \supset ((A > \Diamond_> C) \leftrightarrow \Diamond_>(A \wedge C)))$ Conjunctivitis, Necessitation
10. $\Diamond_> \Diamond_> (A \wedge C)$ Conjunctivitis, K for $\Diamond_>$, 7,8,9
11. $\Diamond_>(A \wedge C)$ 4 axiom, 10
12. $A > \Diamond_> C$ Conjunctivitis, 1, 11
13. $\Diamond_>(A > C) \supset A > \Diamond_> C$ conditional proof, 6-12
14. $(A > \Diamond_> C) \leftrightarrow \Diamond_>(A > C)$ PL, 5,13
15. $\Diamond_> A \supset ((A > \Diamond_> C) \leftrightarrow \Diamond_>(A > C))$ conditional proof, 1-14

A.2 Deontic modals

Fact 1. $R_O \vdash$ Iffy Ought Thesis

1. $\mathcal{O}^A B$ assumption
2. $A > \mathcal{O}^A B$ 1, NSD
3. $A > \Box_> A$ Persistence, def. of $\Box_>$
4. $A > \mathcal{O}^\top A$ Must entails ought, Agglomeration, 3
5. $A > \mathcal{O}^A \top$ O-Normality, Agglomeration
6. $A > \mathcal{O}^\top B$ O-CSO 2,4,5
7. $\mathcal{O}^A B \supset A > \mathcal{O}^\top B$ conditional proof, 1-6

8. $A > \mathcal{O}^{\top} B$ assumption

9. $A > \mathcal{O}^{A} \top$ O-Normality, Agglomeration

10. $A > \Box_{>} A$ Persistence

11. $A > \mathcal{O}^{\top} A$ Persistence, def. of $\Box_{>}$

12. $A > \mathcal{O}^{A} B$ O-CSO, 8,9,11

13. $\mathcal{O}^{A} B$ NSD, 12

14. $A > \mathcal{O}^{\top} B \supset \mathcal{O}^{A} B$ conditional proof, 8-13

15. $\mathcal{O}^{A} B \leftrightarrow A > \mathcal{O}^{\top} B$ 7,14

B Soundness results

B.1 Soundness for R

Define a shifty frame as a triple $\langle R, P, f\rangle$ such that for some set of worlds W:

- $R \subseteq W \times W$ and R is reflexive
- $P = \{\langle w, R'\rangle : w \in W, R' \subseteq R\}$
- $f : \mathcal{P}(P), P \mapsto \mathcal{P}(P)$

P contains the points of evaluation, world-accessibility relation pairs, where the worlds are drawn from some given set W and the accessibility relations are a subrelation of R. For any given point $p \in P$, say that p_w is the world-coordinate of p and p_R is the accessibility relation coordinate. f is the selection function. I write $f_w(\mathbf{A}, \langle w, R\rangle)$ for $\{w' : \exists p \in f(\mathbf{A}, \langle w, R\rangle) : w' = p_w\}$ and $f_R(\mathbf{A}, \langle w, R\rangle)$ for $\{R' : \exists p \in f(\mathbf{A}, \langle w, R\rangle) : R' = p_R\}$.

A shifty model adds to a frame a valuation function, mapping atomics to $\mathcal{P}(W)$. Booleans have their usual semantics:

Atomics. $\langle w, R\rangle \models p$ iff $w \in V(p)$

Negation. $\langle w, R\rangle \models \neg A$ iff $\langle w, R\rangle \not\models A$

Conjunction. $\langle w, R\rangle \models A \wedge B$ iff $\langle w, R\rangle \models A$ and $\langle w, R\rangle \models B$

Disjunction. $\langle w, R\rangle \models A \vee B$ iff $\langle w, R\rangle \models \neg(\neg A \wedge \neg B)$

Where $\mathbf{A}$ is a set of points from P, define $\downarrow\mathbf{A}$ as $\{w \in W\text{: for some } p \in \mathbf{A}\ w = p_w\}$.

We give the following entry for the conditional.

Restricting Operator Semantics. $\langle w, R\rangle \models A > C$ iff for all $\langle w', R'\rangle \in f([\![A]\!], \langle w, R\rangle)$: $\langle w', R'\rangle \models C$

Validity is preservation of truth at proper points $\langle w, R\rangle$, points where $w \in R(w)$:

Definition of validity.. $A_1, ..., A_n \models C$ iff for all proper $\langle w, R\rangle$ if $\langle w, R\rangle \models A_1, ..., \langle w, R\rangle \models A_n$ then $\langle w, R\rangle \models C$.

As usual, $\models A$ iff $\varnothing \models A$.

We add the following conditions on the selection function. Recall that $R^{\mathbf{A}} = max\{R' \subseteq R : \text{if } wR'w' \text{ then } \langle w', R'\rangle \in \mathbf{A}\}$.

Propriety. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' \in R'(w')$.

Update Constraint. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $R' \in R^{\mathbf{A}}$.

Visibility. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' \in R(w)$

Non-Vacuity. If for some $R' \in R^{\mathbf{A}}$ $R'(w) \neq \varnothing$, then $f(\mathbf{A}, \langle w, R\rangle) \neq \varnothing$

CMon. If $f(\mathbf{A}, \langle w, R\rangle) \subseteq \mathbf{B}$, then $f_w(\mathbf{A} \cap \mathbf{B}, \langle w, R\rangle) \subseteq f_w(\mathbf{A}, \langle w, R\rangle)$

Union. $f_w(\mathbf{A} \cup \mathbf{B}, \langle w, R\rangle) \subseteq f_w(\mathbf{A}, \langle w, R\rangle) \cup f_w(\mathbf{B}, \langle w, R\rangle)$

Flat. When $\mathbf{A}$ and $\mathbf{B}$ are non-modal $f(\mathbf{A} \cap \mathbf{B}, \langle w, R\rangle) = f(\mathbf{A} \cap \mathbf{B}, \langle w', R'\rangle)$, for all $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$.

Relational Uniqueness. If $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ and $\langle w'', R''\rangle \in f(\mathbf{A}, \langle w, R\rangle)$, then $R' = R''$ and $R'(w') = R'(w'')$.

Say that a shifty frame that obeys all of the above constraints is a *basic* shifty frame.

Fact 1. If A is Boolean, then $w, R \models A$ iff $w, R' \models A$.
Proof. This follows trivially from the entries for atomics and the Boolean connectives.

Fact 2. If A is Boolean, then $R^{[\![A]\!]} = \{R \cap (W \times \downarrow[\![A]\!])\}$.
Proof. First note that $R \cap (W \times \downarrow[\![A]\!])$ is such that if $\langle w_1, w_2\rangle \in R \cap (W \times \downarrow[\![A]\!])$ then $w_2, R \cap (W \times \downarrow[\![A]\!]) \models A$. Because in general $\downarrow[\![A]\!]$ is the set of worlds w such that for some R' $w, R' \models A$; but by Fact 1, since A is Boolean, $w, R' \models A$ iff $w, R'' \models A$ for any R''.

And clearly $R \cap (W \times \downarrow[\![A]\!])$ is maximal: for any $R' \in R^{[\![A]\!]}$ if $\langle w_1, w_2\rangle \in R'$ then $\langle w_1, w_2\rangle \in R$, $w_2 \in \downarrow[\![A]\!]$ and so $w_2, R \cap (W \times \downarrow[\![A]\!]) \models A$. By similar reasoning $R \cap (W \times \downarrow[\![A]\!])$ must be the unique member of $R^{[\![A]\!]}$.

Thus, when A is Boolean, I adopt the convention using "$R^{[\![A]\!]}$" for the unique member of $R^{[\![A]\!]}$.

Fact 3. If $\mathcal{F}$ is a basic shifty frame, then R is sound on $\mathcal{F}$.

Proof. First we prove that the inference rules of R are sound on $\mathcal{F}$. It's trivial to show that Detachment and LLE preserve validity. Given Propriety, Agglomeration preserves validity: validity is defined on proper points and Propriety ensures that if $\langle w', R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $\langle w', R'\rangle$ is proper.

Now we prove the axioms are valid.

- Identity. Note that by the Update Constraint and Propriety, if $\langle w', R'\rangle \in f([\![A]\!], \langle w, R\rangle)$ then $\langle w', R'\rangle \in [\![A]\!]$: for by the Update Constraint if $w'R'w''$ then $\langle w'', R'\rangle \in [\![A]\!]$ and by Propriety $w'R'w'$.

- Simple Cautious Monotoncity. Let C be Boolean and $\langle w, R\rangle$ proper. Suppose $w, R \models A > B$ and $w, R \models A > C$. Thus $f([\![A]\!], \langle w, R\rangle) \subseteq [\![B]\!]$ and $f_w([\![A]\!], \langle w, R\rangle) \subseteq\downarrow[\![C]\!]$. By CMon, $f_w([\![A]\!] \cap [\![B]\!], \langle w, R\rangle) \subseteq f_w([\![A]\!], \langle w, R\rangle)$. Thus $f_w([\![A]\!] \cap [\![B]\!], \langle w, R\rangle) \subseteq\downarrow[\![C]\!]$, ie $f_w([\![A \wedge B]\!], \langle w, R\rangle) \subseteq\downarrow[\![C]\!]$. Since C is Boolean, by Fact 1, if $w' \in f_w([\![A \wedge B]\!], \langle w, R\rangle)$ then for all R' $w', R' \models C$; thus $w, R \models (A \wedge B) > C$.

- Simple OR. Let C be Boolean and $\langle w, R\rangle$ proper. Suppose that $w, R \models A > C$ and $w, R \models B > C$. Then by Union $f_w([\![A]\!] \cup [\![B]\!], \langle w, R\rangle) \subseteq f_w([\![A]\!], \langle w, R\rangle) \cup f_w([\![B]\!], \langle w, R\rangle)$. By assumption $f_w([\![A]\!], \langle w, R\rangle) \cup f_w([\![B]\!], \langle w, R\rangle) \subseteq\downarrow[\![C]\!]$. Thus $f_w([\![A]\!] \cup [\![B]\!], \langle w, R\rangle) \subseteq\downarrow[\![C]\!]$ and, by Fact 1, $f([\![A \vee B]\!], \langle w, R\rangle) \subseteq [\![C]\!]$.

- Simple Persistence. Let A be Boolean. Trivially $w, R \models A > (B > A)$ if $f([\![A]\!], \langle w, R\rangle) = \varnothing$. So suppose $\langle w_1, R_1\rangle \in f([\![A]\!], \langle w, R\rangle)$. Again trivially $\langle w_1, R_1\rangle \models B > A$ if $f([\![B]\!], \langle w_1, R_1\rangle) = \varnothing$. So suppose $\langle w_2, R_2\rangle \in f([\![B]\!], \langle w_1, R_1\rangle)$. By Fact 2 and the Update Constraint, $R_1 = R \cap (W \times \downarrow[\![A]\!])$ and by Visibility $w_1 R_1 w_2$. Thus, since A is Boolean $w_2, R_2 \models A$ and thus $f([\![B]\!], \langle w_1, R_1\rangle) \subseteq [\![A]\!]$.

- Boolean Flattening. Let A, B, C be Boolean. $\langle w, R\rangle \models A > (A \wedge B) > C$ iff $f([\![A]\!], \langle w, R\rangle) \subseteq [\![(A \wedge B) > C]\!]$ iff for all $\langle w', R'\rangle \in f([\![A]\!], \langle w, R\rangle)$ $f([\![A \wedge B]\!], \langle w', R'\rangle) \subseteq [\![C]\!]$ iff, since A and B are Boolean, by Flattening $f([\![A \wedge B)]\!], \langle w, R\rangle) \subseteq [\![C]\!]$ iff $\langle w, R\rangle \models (A \wedge B) > C$.

- Modal CEM. Suppose that Modal CEM is not valid: then for some point $A > (B > \bot)$ and $A > \neg(B > \bot)$ are both false. Thus for some $\langle w_1, R_1\rangle, \langle w_2, R_2\rangle \in f([\![A]\!], \langle w, R\rangle)$ $w_1, R_1 \not\models B > \bot$ and $w_2, R_2 \not\models \neg(B > \bot)$ ie $w_2, R_2 \models B > \bot$. To show Relational Uniqueness fails, we assume $R_1 = R_2$ and show $R_1(w_1) \neq R_1(w_2)$. First notice there is some $\langle w_3, R'\rangle \in f([\![B]\!], \langle w_1, R_1\rangle)$, where $R' \subseteq R_1$, by the Update Constraint and $w_1 R_1 w_3$ by Visibility. But if it were true that $R_1(w_1) = R_1(w_2)$, then it would be that $w_2 R_1 w_3$; and since by the Update Constraint $w_3, R' \models B$, $w_2 R' w_3$. Thus, by Non-Vacuity, $f([\![B]\!], \langle w_2, R_1\rangle) \neq \varnothing$; but this contradicts our assumption that $w_2, R_2 \models B > \bot$.

Fact 4. Identity of Accessibility is valid on basic shifty frames.

Proof. Let A be Boolean. Suppose first $w, R \models \Box A$. Then for all $w' \in R(w) : w', R \models A$. Thus $R^{[\![\neg A]\!]}(w) = \varnothing$. Now suppose $w, R \not\models \neg A > \bot$. Then there is some $\langle w', R'\rangle \in f([\![\neg A]\!], \langle w, R\rangle)$. By Fact 3, $w', R' \models \neg A$ and since $\neg A$ is Boolean $w', R \models \neg A$. But by Visibility, wRw' which contradicts our assumption.

Now suppose $w, R \models \neg A > \bot$. By Non-Vacuity, $R^{[\![\neg A]\!]}(w) = \varnothing$. But since $\neg A$ is Boolean, $R^{[\![\neg A]\!]} = R \cap (W \times \downarrow [\![\neg A]\!])$. Thus, by Fact 1, if wRw' then $w', R \models A$.

Footnote 23 asserts the following condition suffices for Boolean Strong Centering:

Minimality. If $\exists R' \in R^{\mathbf{A}}$ such that $\langle w, R'\rangle \in \mathbf{A}$ then $\exists R' \in R^{\mathbf{A}}$ such that $\langle w, R'\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ and if $\langle w', R''\rangle \in f(\mathbf{A}, \langle w, R\rangle)$ then $w' = w$.

Fact 5. If Minimality holds on $\mathcal{F}$ then Boolean Strong Centering is valid on $\mathcal{F}$.

Proof. Suppose $\langle w, R'\rangle \models A$ and C is Boolean. Then by Minimality and Relational Uniqueness, $f([\![A]\!], \langle w, R\rangle) = \{\langle w, R^{[\![A]\!]}\rangle\}$. By Fact 1, $w, R \models C$ iff $w, R^{[\![A]\!]} \models C$.

B.2 Soundness for R_O + *NSD*

Say that a shifty deontic frame is a shifty frame $\mathcal{F}$ together with an ordering function $\leq$. We assume the definitions from the previous section are carried over in the natural way. The semantics for $\mathcal{O}$ are as in the main text.

Now let $R+$ be the logic that results from adding to R, O-Normality, O-MOD and O-CSO.

Fact 1. If $R+$ is valid on a deontic shifty frame $\mathcal{F}$, then NSD is valid on $\mathcal{F}$ only if Tracking is.

Proof. Suppose $R+$ is valid on $\mathcal{F}$ but Tracking fails. Then, for some non-modal $\mathbf{A}$, there is some w_1 and w_2 such that $w_2 \in f(\mathbf{A}, \langle w_1, R\rangle)$ but either there is some $w_3 \in R(w_1) \cap \downarrow\mathbf{A}$ and $w_3 \notin R(w_2) \cap \downarrow\mathbf{A}$ or there is some $w_3 \in R(w_2) \cap \downarrow\mathbf{A}$ and $w_3 \notin R(w_1) \cap \downarrow\mathbf{A}$.

First take the case where $w_3 \in R(w_1) \cap \downarrow\mathbf{A}$ but $w_3 \notin R(w_2) \cap \downarrow\mathbf{A}$. Suppose that $R(w_2) \cap \mathbf{A}$ is empty. Then $\mathcal{B}(w_2, R^{\mathbf{A}})$ is empty, while $\mathcal{B}(w_1, R^{\mathbf{A}})$ is not. Thus, letting $V(p) = \downarrow\mathbf{A}$, we have $\langle w_2, R\rangle \models \mathcal{O}^p \bot$. But by Modal CEM, for any other w_4 if $\langle w_4, R'\rangle \in$

$f(\mathbf{A}, \langle w_1, R\rangle)$ then $R' = R\cap \downarrow\mathbf{A}$ and $R(w_4)\cap \downarrow\mathbf{A} = R(w_2)\cap \downarrow\mathbf{A}$. Thus $w_4, R^{\mathbf{A}} \models \mathcal{O}^p\bot$ for all $w_4 \in f_w(\mathbf{A}, \langle w_1, R\rangle)$; thus $w_1, R \models p > \mathcal{O}^p\bot$ but $w_1, R \not\models \mathcal{O}^p\bot$.

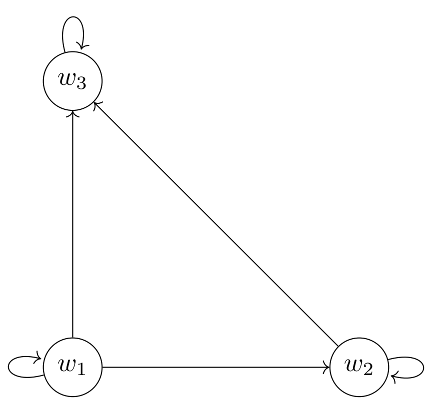
Now assume $R(w_2)\cap \downarrow\mathbf{A}$ is not empty. Take a frame $\mathcal{F}'$ where W and R are the same as in $\mathcal{F}$ but our ordering $\leq$ is such that $\mathcal{B}(w_1, R^{\mathbf{A}}) = \{w_3\}$. Now build a model on $\mathcal{F}'$ such that $V(p) = \downarrow\mathbf{A}$ and $V(q) = \{w_3\}$. We have $\langle w_1, R\rangle \models_{\mathcal{F}',V} \mathcal{O}^p q$. But we also have $w_2, R^{\mathbf{A}} \models_{\mathcal{F}',V} \mathcal{O}^p\neg q$ and since in this case $\mathcal{B}(w_2, R^{\mathbf{A}})$ is not empty by assumption, $w_2, R^{\mathbf{A}} \models_{\mathcal{F}',V} \neg\mathcal{O}^p q$. Thus $\langle w, R\rangle \not\models_{\mathcal{F}',V} (p > \mathcal{O}^p q)$.

Now suppose $w_3 \in R(w_2)\cap \downarrow\mathbf{A}$ and $w_3 \notin R(w_1)\cap \downarrow\mathbf{A}$. Note that, by Visibility, $R(w_1)\cap \downarrow\mathbf{A}$ cannot be empty in this case. Take a frame $\mathcal{F}'$ where W and R are the same as in $\mathcal{F}$ but our ordering $\leq$ is such that $w_3 \in \mathcal{B}(w_2, R^{\mathbf{A}})$. Let $V(q) = \{w_3\}$. Now we have $w_1, R \models_{\mathcal{F}',V} \mathcal{O}^p\neg q$. But we also have $w_2, R^{\mathbf{A}} \not\models_{\mathcal{F}',V} \mathcal{O}^p\neg q$ and so $w_1, R \not\models_{\mathcal{F}',V} p > \mathcal{O}^p\neg q$

Fact 2. On any deontic shifty frame that validates $R+$, if Tracking and Supervenience hold, then NSD is valid for Boolean A, C.

Proof. Suppose $\mathcal{F}$ is an $R+$ frame where Tracking and Supervenience hold. We show $\mathcal{B}(w, R^{[\![A]\!]}) = \mathcal{B}(w', R^{[\![A]\!]})$, for all $w' \in f([\![A]\!], \langle w, R\rangle)$. Since A is Boolean, we have $R^{[\![A]\!]} = R \cap (W\times \downarrow[\![A]\!])$. By Tracking, when $w' \in f([\![A]\!], \langle w, R\rangle)$, $R(w)\cap \downarrow[\![A]\!] = R(w')\cap \downarrow[\![A]\!]$, i.e. $R^{[\![A]\!]}(w) = R^{[\![A]\!]}(w')$. But then given Supervenience, it's trivial that $\mathcal{B}(w, R^{[\![A]\!]}) = \mathcal{B}(w', R^{[\![A]\!]})$.

Fact 3. There are frames where Tracking holds but R is not symmetric.
Proof. Recall:



Say that, in a given shifty frame, where s is a set of worlds $\uparrow s = \{\langle w, R\rangle : w \in s$ and $R \in P\}$. In the above frame, the non-modal propositions are $\top, \bot, \uparrow\{w_i\}$ for $1 \leq i \leq 3$, and:

- $\mathbf{A} = \uparrow\{w_2, w_3\}$
- $\mathbf{B} = \uparrow\{w_1, w_3\}$

- $\mathbf{C} = \uparrow\{w_1, w_2\}$

We now specify a selection function for our frame above. Since Tracking is restricted to non-modals, we specify f for non-modals at proper points. Assume that for any non-modal $\mathbf{D}$:

- $f_w(\mathbf{D}, \langle w_i, R' \rangle) \leq 1$
- $f_w(\mathbf{D}, \langle w_i, R' \rangle) \subseteq \downarrow \mathbf{D}$
- If $w_i \in \mathbf{D}$ then $f_w(\mathbf{D}, \langle w_i, R' \rangle) = \{w_i\}$
- If $f_w(\mathbf{D}, \langle w_i, R' \rangle) = \{w_j\}$ then $w_i R' w_j$.
- If $f_w(\mathbf{D}, \langle w_i, R' \rangle) = \varnothing$, then $\mathbf{D} \cap R'(w_i) = \varnothing$

In addition, assume $f_w(\mathbf{A}, \langle w_1, R \rangle) = \{w_2\}$. The reader can verify that these conditions uniquely pin down the selected world for any non-modal **D**D at any proper point. Since we only care about non-modals, the shift in accessibility relations is easily specified: $f_R(\mathbf{D}, \langle w_i, R' \rangle) = (R')^{\mathbf{D}}$. Thus $f(\mathbf{D}, \langle w_i, R' \rangle)$ is recoverable from f_w and f_R.

Fact 4. If Tracking holds on a basic shifty frame, then R is transitive.
Proof. Suppose R is not transitive. Then for some w_1, w_2 and w_3, $w_1 R w_2$, $w_2 R w_3$ but not $w_1 R w_3$. Let $V(p) = \{w_2, w_3\}$. By Non-Vacuity and Visibility, $f_w([\![p]\!], \langle w_1, R \rangle) = \{w_2\}$. $w_3 \in R(w_2) \cap V(p)$ but $w_3 \notin R(w_1) \cap V(p)$.

Fact 5. If Tracking holds on a basic shifty frame, then R is connected.
Proof. Suppose R is not connected: then, for some w_0, w_1, w_2: $w_0 R w_1$ and $w_0 R w_2$ but neither $w_1 R w_2$ nor vice versa. Now let $V(p) = \{w_2, w_3\}$. By Propriety and the Update Constraint $f_w([\![p]\!], \langle w, R \rangle) \subseteq \{w_2, w_3\}$. And thus for all $w' \in f([\![p]\!], \langle w, R \rangle)$ $R(w') \cap V(p) \neq R(w_0) \cap V(p)$.

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