Persistence is Trivial

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Abstract

This paper demonstrates a conflict between two intutions about the indicative conditional. On the entailment intuition, the logic of the indicative shares the structural features of an entailment relation. On the restrictor picture, conditionals shift the interpretation of material in their consequents. I show the space of tenable theories implementing both pictures, at least to some degree, is very small. I regiment the restrictor picture as a commitment to the principle I call Persistence, which says that $A > (C > A)$ is a logical truth. I show that fully general versions of the restrictor intuition are not cotenable with very minimal implementations of the entailment picture; and a minimal version of the restrictor intuition puts a limit on how far the entailment intuition can be developed. I argue a theory of the conditional based on the notion of acceptance achieves the best balance of features from both pictures.

This paper examines two different intuitions that might shape the logic of the indicative conditional. According to the *entailment* intuition, the conditional should share basic structural properties of an entailment relation. According to the *restrictor* intuition, a conditional adds its antecedent to some body of information and thereby *restricts* the interpretation of its consequent.

Examples can be found to support either intuition. Bare indicatives are often congenial to the entailment intuition. Suppose Alice says to Billy:

(1) If we cut the blue wire, bomb A will be disarmed.

(2) If we cut the blue wire, bomb B will be disarmed.

If the conditional mirrors an entailment relation, we would expect it to follow that:

(3) If we cut the blue wire, both bombs will be disarmed.

Just as A entails $B \wedge C$, when A entails B and C individually, so too $A > (B \wedge C)$ follows from $A > B$ and $A > C$.

Right-nested conditionals motivate the restrictor intuition. Suppose Billy demurs:

(4) Well, if the red light is on, then if we cut the blue wire, the bomb will be disarmed.

Billy is saying that the conditional *if you cut the blue wire, the bomb will be disarmed* is true only if we hold fixed that the red light is on. In the context of his utterance, the nested conditional is reinterpreted to only quantify over possibilities where the red light is on.

On the face of it, these are not obviously competing intuitions: one might hope for a theory which implements both. This paper shows the entailment intuition and the restrictor intuition are not in fact jointly tenable in full generality on pain of triviality. In fact, I show that extremely minimal assumptions suffice to generate triviality results, much more minimal than has previously been supposed.

To show this, I first regiment the restrictor picture as a commitment to a logical principle I call *Persistence*, the principle that $A > (C > A)$ is a logical truth. I then prove a number of triviality results: full Persistence, together with a very weak implementation of the entailment intuition, entails the conditional has some of the absurd properties of the material conditional analysis. I conclude the costs of Persistence are too high for it to be valid in full generality.

A weak, and very plausible, form of Persistence, Boolean Persistence, is not obviously impugned by these results. But when we try to combine Boolean Persistence with the entailment intuition, I show the space is still surprisingly constrained: on pain of triviality, Boolean Persistence cannot be added to forms of the entailment view containing the CSO principle, a principle which its leading implementations do validate. In this case, I argue that the entailment view is at fault: Boolean Persistence is much more plausible than the instances of CSO required for this argument.

If we cannot have both, what theory strikes the best balance? I suggest we build a theory of the conditional on Yalcin (2007)'s notion of *acceptance*. Put roughly, an information state *i accepts A* just in case A is true throughout i, when all accessibility relations for any epistemic modal expressions in A are shifted to ones that only range over worlds in i. This notion is naturally coupled with conditionals: evaluating $A > C$ involves moving some information state that accepts A. I argue this kind of theory captures the most plausible parts of both the entailment and the restrictor pictures, while giving satisfying explanations of where both pictures fail.

1 The two intuitions

I'll start by saying more to flesh out the entailment intuition and the restrictor intuition. I then attempt to regiment the restrictor picture axiomatically.

1.1 The intuitive contrast

On the entailment intuition, the indicative should mirror the structural features of some kind of entailment relation. Many of the classic theories of the indicative theories capture this intuition precisely by taking the conditional express a kind of *sui generis* entailment relation: for $A > C$ to be true is for A *in some sense* to entail C. On a traditional strict conditional view, the relevant notion of entailment is simply classical entailment given our evidence: $A > C$ says that the conjunction of one's evidence with A entails C. On a traditional variably strict view, the relevant notion of entailment is stated by appeal to closeness: $A > C$ says that the closest worlds where A holds entails C.[1] This latter view is particularly congenial to the entailment intuition if we think of the indicative as expressing a kind of defeasible, *non-monotonic* inference relation, where $A > C$ encodes willingness to infer C from A.[2]

The restrictor intuition says conditional antecedents shift a body of information held fixed by the consequent, adding to it the antecedent of the conditional. To get a more general sense of the picture, consider some examples:[3]

(5) If the die landed on a number between 1 and 3, then if it landed on an even number, it landed on 2.

(6) If a Republican wins, then if Reagan doesn't win, Anderson will.

[1]The classic strict and variably strict accounts here are those of Lewis (1912) and Stalnaker (1975, 2014), respectively. See also the more recent accounts of Rothschild (2011), Mandelkern (2021, forthcoming).

[2]See Kraus et al. (1990) and Makinson (1994) for the basic choice points here.

[3](5) is from Goldstein and Santorio (2021) and (6) is from McGee (1985).

(5) seems obvious: the conditional in the consequent holds fixed that the die landed between 1 and 3. (6) is a truism, at least to psephologists: the consequent holds fixed that a Republican wins; and since Reagan and Anderson were the two Republicans that year, the whole conditional is true. According to the restrictor picture, this effect is part of the semantics of the conditional: the information held fixed is *shifted* by the conditional; that is why the consequent conditional holds fixed the antecedent.

Restrictor behaviour is in evidence too when we consider epistemic modals in the consequents of conditionals. Consider:[4]

(7) If that is Venice Beach, then we can't be in Kansas.

(8) If that is Venice Beach, then we might be near LAX.

In both the antecedent restricts the domain of the modal. For instance, (7) makes no commitments about what our evidence *unconditionally* entails; rather it says that in all worlds compatible with what we know *and where we are near Venice Beach*, we are not in Kansas. Again, the restrictor picture says that it is the semantics of the conditional that shifts the modal's domain.

1.2 Regimenting the intuitions

To do anything with these intuitions, we must somehow regiment them. I propose to regiment the restrictor intuition as the principle I call *Persistence*.[5]

Persistence. $A > (C > A)$

Persistence is independently plausible. Attempts to deny the principle or hedge on its truth simply seem confused:

(9) #If it's raining, then it's not the case that if it is cold, then it is raining.

(10) #If it's raining, then if it's cold, then maybe it's not raining.

But Persistence also captures the intuitive spirit of the restrictor picture: a right-nested conditional holds fixed the initial antecedent. It thus explains the patterns exemplifying the restrictor view. Return to the McGee example, (6). Persistence tells us that the following is valid.

[4] See von Fintel, Kai and Irene Heim (2021) for further argument.

[5] I use > for the indicative conditional; I use upper case letters for variables over atomics; and I use lower case for atomics.

(11) If a Republican wins, then if Reagan doesn't win, a Republican wins.

The following is also plausibly valid:

(12) If a Republican wins, then if Reagan doesn't win, Reagan doesn't win.

And together, (11) and (12) entail (6).

Persistence also explains the epistemic modal data, given independent relationships between epistemic modals and conditionals. Following Dorr and Hawthorne (ms.), it is natural to identify epistemic necessity claims with certain conditional claims. We might take *must* A to simply be equivalent to $\neg A > \bot$:

Identity of Accessibility. *must* $A \leftrightarrow (\neg A > \bot)$

Standardly, $\neg A > \top$ is true only when its antecedent is in some sense impossible. Identity of Accessibility stipulates that this sense is epistemic.

Given a weak background logic, Persistence entails a principle I call Modal Persistence:[6]

Modal Persistence. $A >$ *must* A

Given Identity of Accessibility, Modal Persistence explains the restrictor data for epistemic modals. Recall:

(7) If we are near Venice Beach, then we must not be in Kansas.

Modal Persistence gives us:

(13) If we are near Venice Beach, then we must be near Venice Beach.

And if it's epistemically necessary we're near Venice Beach, then it's epistemically necessary that we're not in Kansas.

I claim Persistence regiments the restrictor intuition. But you might think the restrictor intuition is better regimented *semantically*. There are variety of semantic accounts which implement restrictor intuition to some degree. There is Kratzer (1977, 2012)'s restrictor account, McGee (1985, 1989)'s variation on Stalnaker's semantics, various dynamic accounts of indicative conditionals, like

[6]Proof sketch: assume Persistence; this gives us $A > (\neg A > A)$; from Identity and Consequent Agglomeration, we have $A > (\neg A > \neg A)$; using Consequent Agglomeration again we can get $A > (\neg A > \bot)$ and from Identity of Accessibility we get $A >$ *must* A. Adding MOD, introduced in §2, makes Persistence and Modal Persistence equivalent.

Gillies (2009) and Willer (2017), and the informational accounts of Yalcin (2007) and Kolodny and MacFarlane (2010).[7]

Semantic accounts don't allow us to answer the central question, which is to what extent the restrictor and entailment intuitions can be maintained together. As it happens, none of these views *fully* validate Persistence: the principle fails for certain sentences with conditional antecedents. While this is of course hints at a tension in the restrictor intuition, it could just as well be attributed to a weakness in our theorising. There is no obvious *theory-neutral* reason to think that Persistence should fail: at this point, Persistence failures in existing accounts might well reflect the limitations of our current technology, rather than well-motivated choices. An account that fully validated Persistence would be preferable to what is already on offer: all things being equal, a theory that implements the restrictor intuition in full generality would be simpler and stronger than a theory which does not.

Alternatively, you might think there is a better axiomatic regimentation of the restrictor intuition, namely Import-Export. Import-Export is the following principle:[8]

Import-Export. $(A \wedge B) > C \leftrightarrow A > (B > C)$

Besides concerning right-nested conditionals, there are obvious affinities between Import-Export and Persistence. Import-Export is validated by many restrictor type views.[9] And Import-Export could also be used to explain the data we have observed.

Nonetheless, I submit that Persistence provides a better precisification of the restrictor intuition. But Import-Export is a strictly stronger principle, given the weak background logic I will assume in §3. It's relatively easy to derive Persistence from Import-Export in this setting: Identity and Consequent Closure give us $(A \wedge C) > A$ and Import-Export then gives us Persistence. In Fact 1 of Appendix A, I show that, unsurprisingly, Import-Export is strictly stronger. Since Persistence alone captures the data above, the extra strength of Import-Export is not needed.

[7]Further interesting variations include the theories in Cariani (2019, 2021), Santorio (2022), Goldstein and Santorio (2021) and Ciardelli (2021).

[8]I use $\leftrightarrow$ for the material biconditional.

[9]As Mandelkern (2018) notes, the number of views that *fully* validate Import-Export is actually quite small.

2 Triviality

I now argue that the two intuitions cannot be maintained together, on pain of triviality.

I prove two triviality results from Persistence together with very minimal assumptions that I take to be parts of any version of the entailment view, monotonic or non-monotonic. Together these results allow us to prove the corollary that, given Persistence, the problematic inferences validated by the material analysis can be recreated entirely within a single indicative.

2.1 First result

For my first result, I use only the two background principles. The first is Identity:

Identity. $A > A$

On any notion of entailment deserving of the name, A entails itself. My second assumption is that conditional consequents agglomerate:

Consequent Agglomeration. if $B_1, \ldots, B_n \vdash C$ then $A > B_1, \ldots, A > B_n \vdash A > C$

Whatever model of entailment we take for the indicative, it is plausible that it will be closed under *logical* entailment. I thus take both principles to be core parts of the entailment view. For the first result, I thus assume our logic is propositional logic plus Identity, Consequent Agglomeration and Persistence; call this system ER.[10]

It will be helpful to note two simple facts about this system. I will often cite the single premise case of Agglomeration, which I call *Consequent Closure*:

Consequent Closure. If $\vdash B \supset C$ then $\vdash (A > B) \supset (A > C)$.

Finally, I will sometimes appeal to the principle I call Normality:

Normality. $A > \top$

This follows trivially from Identity and Consequent Closure.

The first triviality result proves the following theorem:

Triviality 1. $\vdash_{ER} \neg(A > B) > (\neg(\neg A > C) > \bot)$

[10] Note that Gillies (2009) evades some triviality results by denying $\neg A$ and $\neg B$ are always logically equivalent whenever A and B are.c. My results do not rely on this rule.

This is perhaps easier to grasp in suppositional terms: supposing two negated conditionals, where the antecedent of one is the negation of the antecedent of the other, leads one into a contradictory state.

Here is a proof sketch. From Persistence, Identity and Consequent Agglomeration we can prove:

A. $\neg(\neg A > C) > (A > (\neg A > \bot))$

But in general, $\neg(A > C)$ entails $\neg(A > \bot)$, given Consequent Agglomeration. So from Persistence and Consequent Agglomeration we can also prove:

B. $\neg(\neg A > C) > (A > \neg(\neg A > \bot))$

Applying Agglomeration to A and B we derive:

C. $\neg(\neg A > C) > (A > \bot)$

Then using the principle that if $\vdash C$ then $\vdash A > C$ gives us:

D. $\neg(A > B) > (\neg(\neg A > C) > (A > \bot))$

However, from another instance of Persistence, plus the fact that $\neg(A > C)$ entails $\neg(A > \bot)$, we also have:

E. $\neg(A > B) > (\neg(\neg A > C) > \neg(A > \bot))$

We get Triviality 1 by applying Consequent Agglomeration to D and E.

While not easy to parse at first sight, Theorem 1 should in fact be highly objectionable to anyone who rejects the material analysis. One serious problem for the material analysis is its treatment of negated conditionals: it says that $\neg(A > B)$ entails A; but there is no evidence for this entailment and plenty against it:

(14) It's not the case that the building will be destroyed if there is an earthquake; and there may or may not be an earthquake.

Triviality 1 essentially recreates problematic elements of the material analysis of negated conditionals. Triviality 1 says that supposing in sequence some negated conditional $\neg(A > B)$ followed by $\neg(\neg A > C)$ leads to inconsistency. On the material conditional analysis, it is clear why this would be: the first conditional entails $\neg A$ and the second entails A; so supposing both leads to a contradiction. But as we saw this is another of the view's bad predictions. It is in fact perfectly consistent to assert both:

(15) It's not the case that if it rains there will be a picnic and it's not the case that if it doesn't rain, there won't be a picnic.

Those who reject the material analysis should also reject Triviality 1: there is no reason to think that supposing $\neg(A > B)$ followed by $\neg(\neg A > C)$ leaves one in an inconsistent information state.

Another way to dramatise the problem is by appeal to Identity of Accessibility. Given the duality of $\Box$ and $\Diamond$, that principle tells us that $\Diamond A$ is equivalent to $\neg(A > \bot)$. One instance of Triviality 1 is:

(16) $\neg(A > \bot) > (\neg(\neg A > \bot) > \bot)$

Then applying Identity of Accessibility gives us:

(17) $\Diamond A > (\Diamond\neg A > \bot)$

This is a particularly absurd result. $\Diamond A$ and $\Diamond\neg A$ are not inconsistent; so why would supposing them jointly lead to inconsistency? Furthermore there seem like pretty natural, assertable examples of conditionals with the form $\Diamond A > (\Diamond\neg A > C)$. Consider:[11]

(18) If Bob might be in his office then if Bob might not be in his office, then we don't know whether Bob's in his office.

Finally, we can directly consider instances of Triviality 1. Take:

(19) If it's not the case that there will be a picnic if it rains, then if it's not the case that there will be a picnic if it *doesn't* rain, then there will be a picnic.

Triviality 1 suggests such a conditional should be trivially true. This seems wrong: it is a very dubious piece of information. Those who like CEM should be especially wary here. For them, when both antecedents are possible, $\neg(A > C)$ and $\neg(\neg A > D)$ are equivalent to $A > \neg C$ and $\neg A > \neg D$. Thus, when it is possible that it will rain and possible that it will not, (19) above should be equivalent to:

(20) If there won't be a picnic if it rains, then if there won't be a picnic if it doesn't rain, then there will be a picnic.

[11] Thanks to [XXX] for this example.

Again this is highly dubious.

2.2 Second triviality result

The next result uses slightly stronger background assumptions. In addition to Identity and Consequent Agglomeration, I assume the MOD principle.

MOD. $(\neg C > \bot) \supset (A > C)$

Roughly, MOD says that if C indicatively entails $\bot$, then everything indicatively entails $\neg C$.

MOD is naturally taken to be a part of the entailment view. It mirrors the principle that something contradictory is always entailed to be false: if A entails $\bot$ then any B entails $\neg A$. MOD says that indicative entailment shares this feature: if A "indicatively" entails a contradiction then every sentence "indicatively" entails A is false.

In $ER + MOD$, we can then prove the following:

Triviality 2. $\vdash_{ER+MOD} \neg(A > C) > (B > \neg C)$

This says that, given a negated indicative, *any* further supposition entails the negation of the consequent.

Here is a proof sketch. Identity and Agglomeration allow us to prove that if $\vdash C$ then $\vdash A > C$. Thus if $C > (A > C)$ is a logical truth, we can derive for any given B:

A. $B > (C > (A > C))$

Substituting B for $\neg(A > C)$ gives us:

B. $\neg(A > C) > (C > (A > C))$

But Persistence also gives us:

C. $\neg(A > C) > (C > \neg(A > C))$

This allows us to derive

D. $\neg(A > C) > (C > \bot)$

Applying MOD and Consequent Closure to D allows us to derive Triviality 1.

Just like with Triviality 1, Triviality 2 forces objectionable features of the material conditional analysis reappear inside the conditional. The material conditional analysis says $\neg(A > C)$ entails $\neg C$. Again, there is little reason to accept this and plenty to reject it. Consider:

(21) It's not the case that there will be picnic if it rains. But maybe there will still be a picnic (because maybe it won't rain).

Triviality 2 wrongly says the following kind of sentence is tautologous:

(22) If it's not the case that there will be a picnic if it rains, then if my favourite number is seven, then there won't be a picnic.

Remember that $\neg(A > C)$ does not entail that $\neg C$: we may take it to be false that there will be a picnic if it rains, and still allow there may be a picnic. Supposing some further, unrelated proposition makes no difference. Those who reject the material analysis should equally reject Triviality 2.

2.3 A corollary

Importantly, given a further slight strengthening of the background logic, Triviality 1 and 2 allow us to tie the indicative even closer to the material. Consider the principle Contraction:

Contraction. $(A > (A > C)) \supset (A > C)$

Contraction seems extremely plausible: iterating the same antecedent is indeed idle. Adding this principle as an axiom allows us to derive Triviality 3.

Triviality 3. $\vdash_{ER+MOD+CONT} \neg(A > C) > \neg(A \supset C)$

In update terms, supposing a conditional to be false necessarily leaves one in a state where the material conditional is false.

Again we sketch the proof. Against the background logic, Triviality 2 and MOD allow us to prove:

A . $\neg(A > C) > (\neg(A > C) > \neg C)$

Contraction allows us to simplify this to:

B. $\neg(A > C) > \neg C$

Similarly, Triviality 1 and MOD allow us to prove:

C. $\neg(A > C) > (\neg(A > C) > (\neg A > \perp)$

Contraction allows us to simplify this to:

D. $\neg(A > C) > (\neg A > \bot)$

Another round of MOD and Contraction allow us to prove:

E. $\neg(A > C) > A$

Triviality 3 then follows from B and E.

Again, those who reject the material analysis should reject Triviality 3. If we think there is no *logical* entailment from $\neg(A > C)$ to $A \wedge \neg C$, we should be no keener on the idea that there is instead an *indicative* entailment. Just consider:

(23) If it's not the case that there will be a picnic if it rains, then it will rain and there will not be a picnic.

This is clearly not a tautology. The bad inferences validated by the material conditional cannot be improved by simply formulating them as indicatives.

Triviality 3 illustrates further why Triviality 1 and 2 should be rejected: as we saw, Triviality 1 forces us to behave as if $\neg(\neg A > C)$ is ruled out, after an update with $\neg(A > C)$; Triviality 2 forces right-nested conditionals to behave as if C is ruled out, after an update with $\neg(A > C)$; Both of these principles are already very close to the principle that an update with $\neg(A > C)$ leaves us in a state where $A \wedge \neg C$ holds: a full-blown commitment to that principle follows from adding only a single, very plausible principle.

3 What these results show

There is a conflict between Persistence and the entailment view, as I have spelled it out. I argue it is Persistence, and so in turn the restrictor intuition, that cannot be maintained in full generality.

Abandoning Consequent Agglomeration is, to my mind, a no-go. How could one deny that (24) and (25) entail (26)?

(24) If there is a party, Alice will come.

(25) If there is a party, Billy will come.

(26) If there is a party, Alice and Billy will come.

Rejecting this is not obviously better than falling into triviality. And my uses of this principle are not applied to a large set of premises, where we might just be able to stomach a failure of Agglomeration.[12]

One might try to reject Identity. Here there is more theoretical space, for as Mandelkern (2021) has shown, a surprisingly large class of theories give up Identity; indeed, some such as Cariani (2019, 2021) have endorsed giving up certain "junk" instances of Identity in order to maintain Import-Export. Such junk instances tend to involve complex, left nested conditionals, examples which are indeed hard to parse. However, this strategy does not seem a viable escape route to me here.

First, rejecting Identity while accepting Persistence is an unstable combination. If Identity fails then, for some A, $A > A$ can be false. But if Persistence holds, then *for the very same* A, $A > (C > A)$ is valid; indeed $A > (\top > A)$ is valid, even though $A > A$ is not. I cannot imagine assenting to $A > (\top > A)$ while dissenting from $A > A$.

Second, the role of Identity is extremely minimal in my arguments. Identity is not needed for Triviality 2: it is only relied on to show the general fact that if $\vdash C$ then $\vdash A > C$. But to prove this, we could alternatively simply add Normality as an axiom. If the defender of Persistence wants to block the use of this rule, then for *every* logically true B, they will need to deny *any* conditional of the form $\neg(A > C) > B$ is logically true.

The defender of Persistence might reject MOD. This is well-motivated as informational views reject MOD for informational contradictions. This does not get us out of Triviality 1, as MOD is not used there. And this response ultimately does not help in Triviality 2 either, for two reasons.

First, consider how MOD is in fact used: it is needed only to get from step D, reprinted below, to Triviality 2.

D. $\quad \neg(A > C) > (C > \bot)$

Here C need not be an informational contradiction: stipulate it to be an atomic and we will still get a problematic conclusion. A simple form of MOD, restricted to atomics, is enough to cause trouble.

Second, it is not obvious that even D alone is desirable: it suggests that one cannot further revise with C, after an initial update with $\neg(A > C)$. For those who do not think that $\neg(A > C)$ entails $\neg C$, it is not clear why this would be.[13]

[12]See for instance Leitgeb (2012), where Agglomeration fails precisely because high probability does not agglomerate.

[13]Thanks to [XXX] for this point.

Left with no other alternatives, I conclude it is unrestricted Persistence which must be abandoned.

4 Comparison to Gibbard and Mandelkern

There exist already a battery of triviality results for Import Export. The most important of those results can be easily reproduced using Persistence, rather than Import-Export. I show that, unlike my results, none fully establish the fundamental tension between the entailment and the restrictor intuitions.

Dale (1974) and Gibbard (1981) proved a collapse to the material from Import-Export, Modus Ponens, Logical Implication:

Logical Implication. If $A \vdash C$ then $\vdash A > C$

Their basic argument for collapse can be easily adapted to use only Persistence. Modus Ponens already gives us that $A > C$ entails $A \supset C$. For the converse direction, note that Persistence gives us $C > (A > C)$; using Modus Ponens we can derive $C \supset (A > C)$. From Logical Implication, Consequence Agglomeration and the assumption that consequence is classical, we can derive $\neg A > (A > C)$; Modus Ponens then gives us $\neg A \supset (A > C)$.

One problem with this argument is that it relies crucially on Modus Ponens for right-nested conditionals. McGee (1985) makes a strong case that Modus Ponens fails in just this case. Another related problem is that, while Modus Ponens is a natural fit for the entailment view, it is not an inevitable consequence of it either. The entailment theorist might prefer that the conditional merely mirror the *structural* principles of an entailment relation, like Identity and Agglomeration; indeed, in various non-monotonic logics, like the C and P logics of Kraus et al. (1990), Modus Ponens is not valid.

Mandelkern (2021) proves an important strengthening of this result, one that does not appeal to Modus Ponens. He proves a collapse to the material given Identity, Import-Export and the following two principles:

Very Weak Monotonicity. If $\vdash A > A$ then if $A \vdash B$ then $\vdash A > B$

Ad Falsum. $((A > B) \wedge (A > \neg B)) \supset \neg A$

This result can similarly be proved using just Persistence rather than Import

Export.[14]

This result does not establish a fundamental tension either. First of all, Ad Falsum is itself an instance of Modus Tollens and so is quite close to Modus Ponens. Given that Modus Ponens is at best optional to the entailment picture, Modus Tollens should come under some suspicion here too.

Second, Ad Falsum is invalidated by a range of informational semantics; and Mandelkern's proofs crucially involve the very applications of this principle that informational views reject. In particular, Mandelkern uses Ad Falsum to infer $\neg(\neg(A > \neg C) \wedge \neg C)$ from $(\neg(A > \neg C) \wedge \neg C) > (A > C)$ and $(\neg(A > \neg C) \wedge \neg C) > \neg(A > C)$. But the antecedent here will, on such approaches, be treated as a mere informational contradiction, roughly, a sentence which is classically consistent but supported by no information state.[15] The negation of an informational contradiction need not be an informational validity: A might hold on no information state, even while $\neg A$ is consistent with some information states. For this reason, informational views in fact invalidate Ad Falsum for mere informational contradictions. For instance, the semantics in Yalcin (2007) and Goldstein and Santorio (2021) both invalidate this instance of Ad Falsum. But this response, as we have already seen, is not available for my results.

[14] Proof that $A > C \vdash A \supset C$

1. $\neg(A > \neg C) \wedge \neg C > \neg(A > \neg C)$	Identity, Very Weak Monotonicity
2. $\neg(A > \neg C) \wedge \neg C > (A > \neg C)$	Persistence, Very Weak Monotonicity
3. $\neg(\neg(A > \neg C) \wedge \neg C)$	1,2, Ad Falsum
4. $\neg C \vdash (A > \neg C)$	3, classical logic
5. $A > C \wedge \neg C \vdash A > C \wedge A > \neg C$	4, classical logic
6. $A > C \wedge A > \neg C \vdash \neg A$	Ad Falsum
7. $A > C \wedge \neg C \vdash \neg A$	5, 6, classical logic
8. $A > C \wedge A \vdash C$	7, classical logic
9. $A > C \vdash A \supset C$	8, classical logic

Proof that $A \supset C \vdash A > C$

1. $(A \supset C \wedge \neg(A > C)) > \neg(A > C)$	Identity, Very Weak Monotonicity
2. $(A \supset C \wedge \neg(A > C)) > (A > (A \supset C))$	Persistence, Very Weak Monotonicity
3. $(A \supset C \wedge \neg(A > C)) > (A > C)$	2, Very Weak Monotonicity
4. $\neg(A \supset C \wedge \neg(A > C))$	3, Ad Falsum
5. $A \supset C \vdash A > C$	4, classical logic

[15] In other words, the same status that Yalcin (2007) assigns to claims of the form $\Diamond A \wedge \neg A$.

5 Against (strengthening) the entailment picture

At this point, the most promising package combines the entailment intuition with a version of the restrictor picture limited to Boolean antecedents. After explaining why that package is the best motivated so far, I show there is a sharp limit on how far it can be developed.

While full Persistence cannot be fully maintained, a restricted form for just Boolean antecedents, looks quite promising:

Boolean Persistence. $A > (C > A)$, when A, C are Boolean.

Boolean Persistence is well-motivated by our starting examples. Moreover, it is not strong enough to reproduce the triviality results. Triviality 1 and 2 require respectively the following left-nested instances of Persistence:

$$\neg(A > C) > (C > \neg(A > C))$$

$$\neg(A > C) > (\neg A > (\neg A > C))$$

Giving up these instances of Persistence does not seem much of a loss. The natural package then adds Boolean Persistence to our implementation of the entailment view from §2.

While this package is non-trivial, it involves an extremely minimal implementation of the entailment intuition. Can we add other plausible structural properties of entailment to this package?

A particularly property is *cumulativity*. A consequence relation $\mid\!\sim$ is *cumulative* iff, when $\Gamma \mid\!\sim A$, then $\Gamma, A \mid\!\sim B$ iff $\Gamma \mid\!\sim B$. As Makinson (1994) puts it, given a cumulative consequence relation, "we may accumulate our conclusions into our premises without loss of inferential power ... or accumulation of it", a natural property for a consequence relation to have.

If we model the conditional on a cumulative entailment relation, then we would expect the following principles to be valid:

Cautious Monotonicity. $((A > B) \wedge (A > C)) \supset (A \wedge B) > C$

Cautious Transitivity. $((A > B) \wedge ((A \wedge B) > C)) \supset A > C$

Now, as is well-known, given a relatively weak background logic,[16] the conjunction of these two principles is equivalent to the principle CSO:

[16] CK, specifically.

CSO. $((A > B \wedge B > A) \wedge A > C) \supset (B > C)$

Phrased in terms of entailment, CSO says if A and B conditionally entail each other, then A and B conditionally entail the same things. Unsurprisingly, CSO is indeed a theorem of most of the well-known conditional logics. Thus it is natural to explore whether CSO can be added to the entailment view, in combination with Boolean Persistence.

5.1 The trouble with Boolean Persistence

In fact, CSO does not play well even with Boolean Persistence. To see this, start with a case. Suppose that, in the upcoming US election, the third party is overwhelmingly likely to win: they have about a 90% chance of victory, with the Democrat and Republican candidates having each a mere 5% chance.

First note that the Republican loses iff either the Democrat or the third party candidate wins; and the Democrat loses iff either the Republican or the third party candidate wins. This makes the following extremely probable:

(27) If the Republican loses, the third party candidate will win.

(28) If the Democrat loses, the third party candidate will win.

These should strike us as having over 90% chance of being true.

But (27) and (28), together with some trivial conditional logic, give us that the following are each at least as likely:

(29) If the Republican loses, the third party candidate or the Republican wins.

(30) If the Democrat loses, the third party candidate or the Democrat wins.

In other words, the following are extremely likely:

(31) If the Republican loses, the Democrat loses.

(32) If the Democrat loses, the Republican loses.

Now the following instance of Boolean Persistence is a truism.

(33) If the Republican loses, then if third party candidate loses, then the Democrat wins.

After all, there are just three candidates and so one of them must win. Thus, from (31), (32) and (33), CSO should allow us to infer that the following is very likely:

(34) If the Democrat loses, then if the third party candidate loses, the Democrat wins.

But of course (34) is in fact absurd.

We can prove that this tension is more general:

Triviality 4. Suppose that A, B, C are pairwise inconsistent. Then, given CSO, Identity, Consequent Agglomeration and Boolean Persistence,

$$(A \vee B) > B, (B \vee C) > B \models (A \vee B) > \neg B > \bot$$

Theorem 4 arises because CSO conflicts with the restricting function of conditional antecedents. CSO tells us that, if it *happens* to be true that the Democrat loses, if the Republican does and *vice versa*, then we should be freely able to substitute "the Republican loses" with "the Democrat loses" in conditional antecedents. Boolean Persistence says that when we consider conditionals with antecedents like "the Republican loses" or "the Democrat loses", the antecedent should be held fixed by right-nested conditionals. But we do not want a conditional with the antecedent "the Republican loses" to *also* hold fixed that the Democrat loses, even if it *happens* to be true that the Democrat loses, if the Republican does and *vice versa*. While they are conditionally equivalent, the two antecedents restrict in different ways. Triviality 4 dramatises this conflict.

Unlike our previous results, this seems a problem for the entailment intuition. Our previous results appealed to complex, hard-to-assess instances of Persistence; but instances of Boolean Persistence seem on extremely solid footing. Consider again (33):

(33) If the Republican loses, then if third party candidate loses, then the Democrat wins.

This is just a truism about the set-up of the case. CSO seems far more likely to be the offender: (31), (32), (33) and (34) offer a much more prima facie compelling counterexample to CSO.[17]

[17] For other attempted counterexamples to CSO, see Tichý (1976), Ahmed (2011) and Bacon (2012). While interesting, I take the judgements in these cases to be less clear than in mine; see also Walters (2011) for further responses.

5.2 Rejecting Boolean Persistence

To maintain a more full-blooded version of the entailment intuition, we might reject even Boolean Persistence. Following the inspiration of Lewis (1996), we might maintain that Boolean Persistence is invalid, but also say that, because of the pervasive context-sensitivity of conditionals, those counterexamples to Persistence are *elusive*.

On this strategy, Persistence is not valid; but when instances of Persistence are uttered, context shifts to make them true. Take again:

(33) If the Republican loses, then if the third party loses, the Democrat wins.

The thought is that, by uttering this sentence, context shifts so that the innermost conditional holds fixed the information that a Republican loses: once it is uttered, we never interpret (33) in a way where the innermost conditional "forgets" the information that a Republican loses. The same happens when we utter the conditional:

(35) If the Democrat loses, then if the third party loses, the Republican wins.

However, the context where (35) is true may be one where (33) is *false*, and vice versa. This is the sense in which counterexamples to Persistence are elusive: after first asserting (33), then asserting (35), for example, pushes us towards an interpretation that makes it true, even if previously we were in a context where it is false. This is a common response to McGee's counterexamples to Modus Ponens, and the appeal of Import-Export; and more recently, in the form of Mandelkern (2021, forthcoming)'s *bounded* theory of indicative conditionals, we find a highly sophisticated version of this basic strategy.[18]

This view responds to our latest result by saying that (33) and (34) get evaluated in different contexts.[19] (33) gets interpreted as holding fixed the information that a Republican loses. In that context, the problematic (34), repeated below, is in fact true:

(34) If the Democrat loses, then if the third party loses, the Democrat wins.

[18]Some views here say the consequent of a right-nested conditional receives a special interpretation, different from that of the main connective. Mandelkern, on the other hand, takes the conditional to be interpreted univocally; instead, when interpreting sentences like (33), the salient notion of closeness to be one where, for any given epistemically possible world where the antecedent is true, the closest worlds are all ones where the antecedent is true. Both views face the above problem.

[19]Similarly, in the proof of Triviality 4, it denies the two instances of Persistence are true in the same context.

But uttering (34) moves us into a context where the corresponding instance of Persistence, (35), is true; in that context, (34) has the false reading we hear; furthermore, (33) is false in that context. All this allows us to maintain CSO, while capturing the appearances in this case: when everything is interpreted univocally, there is no counterexample to CSO.

The problem, however, is that it seems that we can quantify over the instances of Boolean Persistence driving the problem. Consider:

(36) If a given party lost the election, then the other party won, if the third party candidate didn't win.

(36) commits us to both (33) and (35) at once; we are saying:

(37) For both parties x: if party x loses then if the third party loses then x's rival party wins.

Given CSO, together with (27) and (28), (36) should entail:

(38) If a given party lost the election, then it won the election, if the third party candidate didn't win.

This is of course absurd.

Here our context-shifting strategy does not help. It falsely predicts that (36) cannot be heard as true in the first place; after all, there is not supposed to be a single context in which both of the relevant instances of Persistence are true.[20]

[20] Probably best thing for the elusive theorist to say here is that in (36) the conditional itself can be directly bound by the quantifier: for concreteness, we could assume the conditional comes with a bindable individual variable in its logical form; and the relevant notion of closeness would be a function of the value of this variable. (Thanks to [XXX] for this suggestion.) This would allow the elusive theorist to make (36) true. Its structure would be something like:

(i) For both parties x: (party x loses $>_x$ (the third party loses $>_x$ x's rival party wins)

Thus there would be no single interpretation of the conditional where both instances of Persistence are true: (33) is true on the reading where x is the Republican; (35) is true when x is the Democrat.

But this will disrupt the apparent validity of *other* patterns of inference that have nothing to do with right-nested conditionals, like Agglomeration. Consider:

(ii) Everyone who came to the party was someone who lost the lottery if a ticket was drawn.

It should now follow by Agglomeration that:

(iii) If a ticket was drawn, everyone who came to the party lost the lottery.

If it's true of each person that they lost if a ticket was drawn, then if a ticket was drawn, all of those people lost.

On the strategy above, such inferences will not necessarily be perceived to be valid. (ii) should have a reading where the quantifier binds the conditional:

6 Acceptance conditionals

Neither intuitions can be developed as much as we might have hoped. Full Persistence simply must be rejected, on pain of triviality. And *cumulative* entailment cannot be our model for the entailment intuition, given Boolean Persistence.

Nonetheless, by trial and error, we have isolated a plausible basic logic for the indicative, consisting of at least Identity, Agglomeration, Boolean Persistence and Boolean CSO:

Boolean CSO. $(((A > B) \wedge (B > A)) \wedge (A > C)) \supset (B > C)$ when A, B, C are Boolean.

I take it to be natural to add to this package Left Logical Equivalence below, as well as the usual theorems and rules of propositional logic:

LLE. If $\vdash A \leftrightarrow B$ then $\vdash (A > C) \leftrightarrow (B > C)$

Call the system that adds to a standard axiomatisation of propositional logic LLE, Agglomeration, Identity, Boolean CSO and Boolean Persistence A logic. Can A can be motivated by any natural picture of the indicative that does not succumb to triviality?

In fact, it will be validated by a theory of the conditional based on the notion of *acceptance*.[21] Roughly, a body of information — that is, a set of worlds — accepts a sentence A when A is true relative to every world in that information state *when epistemic accessibility is reinterpreted so that only worlds in that information state are accessible.* Acceptance has structural features that mere entailment does not. For one, acceptance is not persistent: an information state can accept A, even while a strict subset of it rejects A. For example, if my belief state is consistent with rain, then I likely will accept the claim that it might rain: when

(iv) For every x who came to the party: a ticket was drawn $>_x$ x lost the lottery.

But it wouldn't follow that for any particular x the following is true:

(v) a ticket was drawn $>_x$ everyone who came to the party lost.

After all, each instance of (iv) is about a different conditional and thus we cannot agglomerate them to derive (v). In fact, things like the following should have consistent readings, even if we suppose that tickets for the lottery were bought by all and only the people at the party:

(vi) It's possible that everyone who came is someone who lost, if a ticket was drawn; though of course if a ticket was drawn, then somebody at the party won.

[21] Note Identity rules out the closely related *dynamic* view of the conditional: as Mandelkern (2021) shows, the leading dynamic views do not validate Identity.

epistemic accessibility is interpreted to range over my belief worlds, $\Diamond$*rain* is true throughout my belief state. However, a subset of my belief state containing only $\neg$*rain* worlds will certainly not accept this claim: when epistemic accessibility is interpreted to range over only worlds in that subset, $\Diamond$*rain* must fail throughout. Logical entailment does not give rise to this kind of situation: adding further premises does not result in a loss of information.

Acceptance is easily built into the conditional. Many think that the conditional involves some notion of update: to assess $A > C$, we suppose A and assess whether C.[22] But what exactly does supposing A involve? A natural answer is *moving to a state which accepts A*. As Boylan and Schultheis (2022), Goldstein and Santorio (2021) and XXX this approach has already proved fruitful in the epistemology of conditionals.

I have two aims in this final section. First, I sketch an informational semantics which makes good on the idea above, showing that the A logic is sound on this semantics and that my triviality results are blocked. But, second, I aim to develop a fairly minimal informational semantics for capturing the A logic. Existing views build much more structure into the indicative than is needed to capture the A logic; for this reason, they validate some disputable further principles. I aim to articulate a more minimal semantics, one that can capture the basic phenomena in this paper without committing to further, unrelated principles. While I leave the task of proving a complete semantics for A to another day, the theory I defend will come closer to characterising A than existing views in the literature. As I show in §6.2, it thus provides a better base theory to build out on in further theorising about the indicative.

6.1 Variably strict domain semantics

Say that an informational variably strict frame is a pair $\langle W, f\rangle$; W is a set of worlds and f, our selection function, is a function from a world-information state pair and a set of world-information state pairs to a set of world-information state pairs. An informational variable strict model adds a valuation function V which assigns to each atomic some subset of W, our set of worlds.

In defining truth in a model, sentences are evaluated at world-information state pairs. Atomics hold at a point just in case the world parameter is in the set of worlds assigned to that atomic by some valuation function; $\wedge$, $\vee$ and $\neg$ have their usual classical semantics.

[22] This thought, of course, goes back at least to Ramsey (1931).

Atomics $w, i \models p$ iff $w \in V(p)$

Negation. $w, i \models \neg A$ iff $w, i \not\models A$

Conjunction. $w, i \models A \wedge B$ iff $w, i \models A$ and $w, i \models B$

Disjunction. $w, i \models A \vee B$ iff $w, i \models \neg(\neg A \wedge \neg B)$

I give a variably strict semantics for the conditional, stated in terms of the selection function. Intuitively, $f(\mathbf{A}, \langle w, i \rangle)$ outputs the set of closest points to $\langle w, i \rangle$ where $\mathbf{A}$ holds.[23] So, where $[\![A]\!] = \{\langle w, i \rangle : \langle w, i \rangle \models A\}$, we have:

Variably strict domain semantics. $w, i \models A > C$ iff for all $\langle w', i' \rangle \in f([\![A]\!], \langle w, i \rangle)$ $w', i' \models C$

Validity is preservation of truth at proper points: $A_1, ..., A_n \models C$ iff, for all $\langle w, i \rangle$ such that $w \in i$, if $w, i \models A_1$, ..., and $w, i \models A_n$ then $w, i \models C$.

We add some formal constraints to the selection function. Since entailment is defined at proper points, we require that the selection function always outputs proper points:

Propriety. If $\langle w', i' \rangle \in f(\mathbf{A}, \langle w, i \rangle)$ then $\langle w', i' \rangle$ is proper.

We also say that the selection function does not discriminate between propositions that differ only in what *im*proper pairs they contain.

Proper Input. If $\mathbf{A}$ is the maximal proper subset of $\mathbf{B}$, then $f(\mathbf{A}, \langle w, i \rangle) = f(\mathbf{B}, \langle w, i \rangle)$.

The most substantive constraint dictates how the information parameter is selected. In evaluating a conditional at $\langle w, i \rangle$, i will serve as the information held fixed for the conditional; this information should be updated by successive antecedents. This is achieved by the Update Constraint. Where $i \models \mathbf{A}$ iff $\langle w, i \rangle \in \mathbf{A}$, for all $w \in i$:

Update Constraint. If $\langle w', i' \rangle \in f(\mathbf{A}, \langle w, i \rangle)$ then $i' \in i + \mathbf{A}$, where $i + \mathbf{A} = max\{i' \subseteq i : i' \models \mathbf{A}\}$

The update constraint says that the closest points to $\langle w, i \rangle$ where $\mathbf{A}$ is true are always ones whose information parameter accepts $\mathbf{A}$; specifically for $\langle w', i' \rangle$ to be in $f(\mathbf{A}, \langle w, i \rangle)$, i' must be a maximal $\mathbf{A}$-accepting subset of i. Say that a basic frame is one where f obeys Propriety, Proper Input and the Update Constraint.

[23] I use bold variables to range over elements of $\mathcal{P}(W \times \mathcal{P}(W))$.

The Update Constraint and Propriety together enforce the familar constraint that the closest **A**-points are themselves **A**-points.

Success. $f(\mathbf{A}, \langle w, i\rangle) \subseteq \mathbf{A}$.

For if $\langle w', i'\rangle \in f(\mathbf{A}, \langle w, i\rangle)$ then $\langle w', i'\rangle$ makes **A** true: for i' must be a maximal, **A**-accepting state; and $\langle w', i'\rangle$ must be proper.[24]

One final constraint is required for Boolean CSO. Variably strict frameworks usually impose a constraint like the following: if the closest A-worlds are B-worlds and the closest B-worlds are A-world, then the closest A-worlds are the closest B-worlds. We impose the following analogue. Where **A** is a set of world-information state pairs, say that $\downarrow\mathbf{A}$ is the set of worlds that occur as the world parameter of some element of **A**.[25]

Reciprocity. If $f(\mathbf{A}, \langle w, i\rangle) \subseteq \mathbf{B}$ and $f(\mathbf{B}, \langle w, i\rangle) \subseteq \mathbf{A}$, then $\downarrow f(\mathbf{A}, \langle w, i\rangle) = \downarrow f(\mathbf{B}, \langle w, i\rangle)$

We now turn to the logic of this theory. It is easy to see Agglomeration and LLE hold, given Propriety and Proper Inputs. Since Success is derivable from our constraints, Identity is valid. As I show in the Appendix, Boolean Persistence follows from the Update Constraint. Reciprocity guarantees Boolean CSO.

Triviality 1 – 3 all fail in this framework.[26] The arguments for Triviality 1 and 2 both appeal to similar instances of Persistence, respectively:

(39) $\neg(A > C) > (C > \neg(A > C))$

(40) $\neg(A > C) > (A > \neg(A > C))$

Neither of these hold on the acceptance semantics because negated conditionals are not persistent: i might accept $\neg(A > C)$ while some subset of i does not. Triviality 3, relying as it does on Triviality 1 and 2, of course fails too.

We can see that negated conditionals are not persistent by observing that, for Boolean A and C, accepting $\neg(A > C)$ requires accepting a possibility claim: it requires accepting that $A \wedge \neg C$ is possible. To accept $\neg(A > C)$ i must contain

[24]This being the minimal implementation motivate by my discussion, there are a variety of natural extra constraints we might want to add to it. We might also consider:

Minimality. If $\langle w, i'\rangle \in \mathbf{A}$ and $i' \in i + \mathbf{A}$, $\{\langle w, i'\rangle\} \in f(\mathbf{A}, \langle w, i\rangle)$

Non-Vacuity. If $i + \mathbf{A} \neq \{\varnothing\}$ then $f(\mathbf{A}, \langle w, i\rangle) \neq \varnothing$

[25]That is, $\downarrow\mathbf{A} = \{w : \langle w, i\rangle \in \mathbf{A}\}$

[26]See Appendix C for countermodels.

some A-worlds: otherwise, the result of updating i with A would be empty and so $A > C$ would be trivially true throughout i. Now notice that if i contains only $A \wedge C$-worlds, it also cannot accept $\neg(A > C)$: by Minimality $A > C$ would be true at all such worlds. Putting the two together then, accepting $\neg(A > C)$ requires accepting that $A \wedge \neg C$ is possible.

Now we can see why (39) and (40) both fail. Take (39) first. Updating with $\neg(A > C)$ results in a state that contains an $A \wedge \neg C$-world. But if we then update with C we will no longer have a state that accepts $\neg(A > C)$: the resulting state cannot contain any C-worlds and so in particular cannot contain any $A \wedge \neg C$-worlds. Thus, by the time we have updated with both antecedents, we are no longer in an information state that accepts $\neg(A > C)$. Thus (39) can fail: once we move from the selected $\neg(A > C)$ points to the selected C-points, we may reach a point where $\neg(A > C)$ fails, since $\neg(A > C)$ does not remain accepted after updating with C. The same basic point applies to (40) as well. Once we update with the second antecedent A, we are again in an information state that does not accept $\neg(A > C)$; and so once we move from the selected $\neg(A > C)$-points to the selected A-points, we may reach a point which no longer accepts $\neg(A > C)$.

And what of Triviality 4? Recall CSO.

CSO. $((A > B \wedge B > A) \wedge A > C) \supset B > C$

CSO is not valid in full generality: it holds when A, B and C are Boolean, but not when C is a right-nested conditional. The reason is that, even if $A > B$ and $B > A$ are both *true*, updating with A and B may have different effects on the information state exploited by right-nested conditionals.

We can see this at work by returning to the scenario from §5. We argued in that scenario, given Boolean CSO, Identity and Consequent Consequence, the following are both highly likely:

(41) If the Republican loses, the Democrat loses.

(42) If the Democrat loses, the Republican loses.

Boolean Persistence, Identity and Consequent Consequence give us the following as certain:

(33) If the Republican loses, then if third party candidate loses, then the Democrat wins.

Now if CSO is valid for right-nested conditionals, from (33) we can derive the absurd (34).

(34) If the Democrat loses, then if the third party candidate loses, the Democrat wins.

This instance of CSO fails in our semantics. Suppose that we have just three worlds, w_D, w_R and w_3, where the Democrat, the Republican and the third party win, respectively. Let w_3 be the actual world, but say we cannot conclusively rule out any of the three candidates. We can thus think of ourselves as being located at $\langle w_3, i \rangle$, where i treats all three candidates as possible victors.

CSO fails because, even when (31) and (32) hold, the closest point to us where the Republican loses need not be the same as the closest point where the Democrat loses. Let the closest point where the Republican loses to be $\langle w_3, i \cap \textit{Republican loses} \rangle$; here, the third party wins, worlds where the Democrat wins are also still accessible, but no worlds where the Republican wins are accessible. Let the closest point where the Democrat loses be $\langle w_3, i \cap \textit{Democrat loses} \rangle$; here again the third party wins but here the only other accessible worlds are ones where the Republican wins. Both (31) and (32) hold. (33) holds also: at $\langle w_3, \textit{Republican loses} \rangle$, it's true that if the third party candidate loses, then the Democrat wins. But (34) is false: at $\langle w_3, i \cap \textit{Democrat loses} \rangle$ it is false that if the third party candidate loses, the Democrat wins.

6.2 Comparison to other informational views

My semantics has important forerunners. One is the domain semantics for the conditional from Yalcin (2007) and Kolodny and MacFarlane (2010). Another is the path semantics of Goldstein and Santorio (2021) and Santorio (2022). I discuss in an informal way how these views can be seen as special cases of the variably strict domain semantics; and I show how they validate further principles that I do not, principles we might reject.

Start with the simple domain semantics from Yalcin (2007), as modified by Kolodny and MacFarlane (2010). On this view, $A > C$ is true at $\langle w, i \rangle$ just in case the result of updating i to accept $[\![A]\!]$ accepts $[\![B]\!]$. More precisely:

Simple domain semantics. $w, i \models A > C$ iff for all $i' \in i + [\![A]\!]$ for all $w' \in i'$ $w', i + [\![A]\!] \models C$

Here $i + \mathbf{A}$ is as defined above.

Like the variably strict semantics, the simple domain semantics says that conditionals update the information state parameter. It also says conditionals quantify over the entire updated information state:[27] $A > C$ is true iff C is true throughout $i + [\![A]\!]$. This can be recovered by the variably strict semantics if we require that all worlds in the information state are equally close. But the variably strict view leaves room to distinguish between worlds in the information state.

For this very reason, while the variably strict view resolves Triviality 4, the simple domain semantics does not. The simple domain semantics validates the following:

$$(A > B) \supset (A > (\neg B > \bot))$$

But this is not valid, as the discussion from §4 shows. Claims like

(27) If the Democrat loses, the third party candidate will win.

do not entail claims like:

(34) If the Democrat loses, then if the third party candidate loses, the Democrat wins.

We can think the first is very likely, while thinking the second is entirely impossible. The simple domain semantics wrongly says this combination of attitudes is irrational.

Next take path semantics. A path is finite sequence of worlds, all drawn from some finite set W. Sentences are evaluated at paths; focussing just on the case of the conditional, we have:

Path update. $p + A = max\{p' \leq p : \text{for any } p'^{*} \text{ which is a permutation of } p'\, p'^{*} \models A\}$

Path semantics. $p \models A > C$ iff $p + A \models C$

Path semantics is conceptually close to mine. From a given path we can read off a world of evaluation: it is the first world in the sequence. The ordering in a path gives us something like a selection function. Finally, the set of worlds appearing anywhere in the sequence is an information state. Thus paths are formally very close to a tuple of a world, an ordering and an information state.

[27] At least in the simple case where $|i + [\![A]\!]| = 1$.

But paths are highly structured an so path semantics goes beyond what I validate. Here I will focus on its treatment of left nested conditionals.[28] Like many brands of informational semantics, path semantics says to update to acceptance with $A > C$ is to update to acceptance to the corresponding material conditional $A \supset C$. It thus validates the following principle:

Material Antecedents. $(((A > B) > C) \wedge \neg A) \supset C$, for Boolean C

The conditional $(A > B) > C$ is equivalent to $(A \supset B) > C$; and so, if $\neg A$ is true, it follows that $A \supset B$ and thus that C is true, given Modus Ponens for Booleans. This is an instance of the feature above: since updating with $A > C$ only updates with $A \supset C$, if the material conditional is already true at the world of evaluation, we just evaluate C at the original world of evaluation.

Material Antecedents is not obviously valid.[29] Consider the following case from Edgington (1991):

> *Coins.* Earlier today, you picked a coin from a bucket of double-headed and double-tails coins. You may or may not have flipped it at 12pm.

Now consider:

(43) If the coin landed heads if flipped, then it is a double-heads.

(44) If the coin landed tails if flipped, then it is a double-tails.

Both claims appear true, even certain. But this is inconsistent with path semantics. Take a world w consistent with what I know and where the coin is not flipped. Material Antecedents tells us at most one of (43) and (44) can be true at w: if the coin is double-heads then only (43) is true; if double-tails, then only (44) is true. Path semantics says we cannot rationally be certain of both.

[28] Path semantics also adds other principles the A logic, such as a Boolean version of the Flattening principle from Mandelkern (2021):

Boolean Flattening. $(A \wedge B) > C \leftrightarrow A > (A \wedge B) > C$, when A and B are Boolean.

This I take to be more plausible than Material Antecedents. But it is straightforward to add Boolean Flattening to my semantics: see my [XXX] for the details.

[29] We could also deny the conditional is interpreted univocally: Kaufmann (2008), for instance, says the antecedent conditional is interpreted with a different accessibility relation to the rest of the conditional. Those who favour this kind of diagnosis should still favour my semantics over path semantics. In [XXX], I generalise this semantics so that conditionals are evaluated relative to accessibility relations rather than simple information states. It is hard to see how path semantics can be generalised in a similar way.

I suspect this is a genuine counterexample to Material Antecedents, but my main aim is more modest. Material Antecedents seems *separable* from the main concerns of my paper: the A logic does not seem to *commit* us to principles like Material Antecedents. As I'll now show, my variably strict semantics makes good on this thought: it has the resources to invalidate Material Antecedents, suggesting it is a better theory for theorising about the base logic of the indicative.

Consider the following simple model of Coins. Assume there are four worlds:

$$W = \{\neg \mathsf{fd_h}, \neg \mathsf{fd_t}, \mathsf{fd_h}, \mathsf{fd_t}\}$$

In constructing the selection function, I assume the relevant notion of closeness first prioritises a match in whether the coin is double-heads or tails; and next prioritises a match in whether the coin was flipped:

$$\neg \mathsf{fd_h} <_{\neg \mathsf{fd_h}} \mathsf{fd_h} <_{\neg \mathsf{fd_h}} \neg \mathsf{fd_t} <_{\neg \mathsf{fd_h}} \mathsf{fd_t}$$

$$\neg \mathsf{fd_t} <_{\neg \mathsf{fd_t}} \mathsf{fd_t} <_{\neg \mathsf{fd_t}} \neg \mathsf{fd_h} <_{\neg \mathsf{fd_t}} \mathsf{fd_h}$$

$$\mathsf{fd_h} <_{\mathsf{fd_h}} \neg \mathsf{fd_h} <_{\mathsf{fd_h}} \mathsf{fd_t} <_{\mathsf{fd_h}} \neg \mathsf{fd_t}$$

$$\mathsf{fd_t} <_{\mathsf{fd_t}} \neg \mathsf{fd_t} <_{\mathsf{fd_t}} \mathsf{fd_h} <_{\mathsf{fd_t}} \neg \mathsf{fd_h}$$

For simplicity, stipulate that the selection function outputs either a singleton or the empty set. Stipulate that the world parameter in the chosen **A**-pair is never further away from the starting world than any other world in the updated information state.

$$f(\mathbf{A}, \langle w, i\rangle) = \{\langle w', i'\rangle\} \text{ only if for any other world } w'' \in i' \; w' \leq_w w''.$$

These constraints fully determine the output of the selection function for all Booleans, assuming the natural valuation. For instance, let **F** be the set of pairs where the coin is flipped; then $f(\mathbf{F}, \langle \neg \mathsf{fd_h}, W\rangle) = \langle \mathsf{fd_h}, \{\mathsf{fd_h}, \mathsf{fd_t}\}\rangle$.

Our starting information state is just W. The selection function must output the following pairs for $F > H$ at the various worlds in the model:

$$f(F > H, \langle \neg \mathsf{fd_h}, W\rangle) = \{\langle \neg \mathsf{fd_h}, \{\neg \mathsf{fd_h}, \neg \mathsf{fd_t}, \mathsf{fd_h}\}\rangle\}$$

$$f(F > H, \langle \mathsf{fd_h}, W\rangle) = \{\langle \mathsf{fd_h}, \{\neg \mathsf{fd_h}, \neg \mathsf{fd_t}, \mathsf{fd_h}\}\rangle\}$$

$$f(F > H, \langle \neg \mathsf{fd_t}, W\rangle) = \{\langle \neg \mathsf{fd_h}, \{\neg \mathsf{fd_h}, \neg \mathsf{fd_t}, \mathsf{fd_h}\}\rangle\}$$

$$f(F > H, \langle \mathsf{fd_t}, W \rangle) = \{\langle \mathsf{fd_h}, \{\neg\mathsf{fd_h}, \neg\mathsf{fd_t}, \mathsf{fd_h}\}\rangle\}$$

Since the coin is double-headed in every selected world, we can see that (43) will be true at $\langle w, W \rangle$ for every $w \in W$. For $F > T$, the selection function outputs the following pairs:

$$f(F > T, \langle \neg\mathsf{fd_h}, W \rangle) = \{\langle \neg\mathsf{fd_t}, \{\neg\mathsf{fd_h}, \neg\mathsf{fd_t}, \mathsf{fd_t}\}\rangle\}$$

$$f(F > T, \langle \mathsf{fd_h}, W \rangle) = \{\langle \mathsf{fd_t}, \{\neg\mathsf{fd_h}, \neg\mathsf{fd_t}, \mathsf{fd_t}\}\rangle\}$$

$$f(F > T, \langle \neg\mathsf{fd_t}, W \rangle) = \{\langle \neg\mathsf{fd_t}, \{\neg\mathsf{fd_h}, \neg\mathsf{fd_t}, \mathsf{fd_t}\}\rangle\}$$

$$f(F > T, \langle \mathsf{fd_t}, W \rangle) = \{\langle \mathsf{fd_t}, \{\neg\mathsf{fd_h}, \neg\mathsf{fd_t}, \mathsf{fd_t}\}\rangle\}$$

Since the coin is double-tails in every selected world, (44) will be true at $\langle w, W \rangle$ for every $w \in W$. But it is nowhere true that the coin is double-heads and double tails, so Material Antecedents fails.

A Import-Export and Persistence

Fact 1. In PC + Identity and Consequent Agglomeration, Import-Export entails Persistence but not vice versa.

Proof. An official proof that, given Identity and Consequent Agglomeration, I-E entails Persistence can be easily extracted from §2.

To show the converse fails we give a model theoretic argument. A Stalnaker frame contains a set of worlds and a selection function f; a model adds a valuation to a frame; and the semantics for the conditional is given using the selection function: $V(A > C, w) = 1$ iff $f([\![A]\!], w) \subseteq [\![C]\!])$.[30] All of the rules of PC are sound on Stalnaker frames, as is Consequent Agglomeration; see Chellas (1975). Thus on a frame that validates Identity and Persistence, any theorem of the basic system from §2 will be valid. We give an example of such a frame where Import-Export is not valid.

- $W = \{w_1, w_2, w_3\}$
- Selection function:
  - $f(\mathbb{A}, w_1) = \{w_1\}$ if $w_1 \in \mathbb{A}$; otherwise $= \varnothing$

[30] I use $\mathbb{A}$ as a variable over sets of worlds.

– $f(\mathbb{A}, w_2) = \{w_2\}$ if $w_2 \in \mathbb{A}$; otherwise $= \varnothing$

– $f(W, w_3) = w_2$; $f(\{w_1, w_2\}, w_3) = \{w_1\}$; $f(\{w_1, w_3\}, w_3) = \{w_1\}$; $f(\{w_2, w_3\}, w_3) = w_2$; $f(\{w_1\}, w_3) = \{w_1\}$; $f(\{w_2\}, w_3) = \{w_2\}$; $f(\{w_3\}, w_3) = \varnothing$.

Identity is valid: by construction, for any $\mathbb{A}$, $f(\mathbb{A}, w) \subseteq \mathbb{A}$.

Persistence is also valid. For suppose for some valuation $A > (C > A)$ is false at some world. Now Persistence cannot fail for w_1 and w_2. For if $A > (C > A)$ fails at w_1, then $f([\![A]\!], w_1) \neq \varnothing$ and so by construction $f([\![A]\!], w_1) = \{w_1\}$. So then $f([\![C]\!], f([\![A]\!], w_1)) = f([\![C]\!], w_1)$ which is either $\{w_1\}$ or $\varnothing$ and so a subset of $[\![A]\!]$; contradiction. The same argument shows that Persistence cannot fail at w_2. So suppose $A > (C > A)$ fails at w_3. Then $[\![A]\!] \neq \{w_3\}$. So $f([\![A]\!], w_3)$ is either $\{w_2\}$ or $\{w_3\}$. But for $1 \leq i \leq 2$ $f([\![C]\!], w_i) = \{w_i\}$ or $= \varnothing$ and thus $f([\![C]\!], w_i) \subseteq [\![A]\!]$. So in fact $f([\![C]\!], f([\![A]\!], w_3)) \subseteq [\![A]\!]$; contradiction.

Import-Export, however, is not valid. Set $[\![p]\!] = W$ and $[\![q]\!] = \{w_1, w_2\}$ and $[\![r]\!] = \{w_2\}$. At w_3 Import-Export fails: $p > q > r$ is true but $(p \wedge q) > r$ is false.

B Proof of Triviality Results

It's first helpful to note the following lemma:

Lemma 1. If $\vdash C$ then $\vdash A > C$

In words, if C is a theorem, then you can right-nest it in any conditional.

1. $\vdash C$ (assumption)
2. $\vdash A > \top$ (Normality)
3. $\top \vdash C$ (1, classical logic)
4. $\vdash A > C$ (1,3, Consequent Agglomeration)

Note also a further, easy to prove lemma:

Lemma 2. $\neg(A > C) \supset \neg(A > \bot)$

Now recall:

Triviality 1. $\neg(A > B) > (\neg(\neg A > C) > \bot)$

Proof:

1. $\neg(A > B) > (\neg(\neg A > C) > \neg(A > B))$ (Persistence)
2. $\neg(A > B) > (\neg(\neg A > C) > \neg(A > \bot))$ (1, Lemma 2, Consequent Agglomeration)
3. $\neg(\neg A > C) > (A > \neg(\neg A > C))$ (Persistence)
4. $\neg(\neg A > C) > (A > \neg(\neg A > \bot))$ (3, Lemma 2, Consequent Agglomeration)
5. $A > (\neg A > A)$ (Persistence)
6. $A > (\neg A > \neg A)$ (Lemma 1)
7. $A > (\neg A > \bot)$ (5,6, Consequent Agglomeration)
8. $\neg(\neg A > C) > (A > (\neg A > \bot))$ (7, Lemma 1)
9. $\neg(\neg A > C) > (A > \bot)$ (4,8, Consequent Agglomeration)
10. $\neg(A > B) > (\neg(\neg A > C) > (A > \bot))$ (9, Lemma 1)
11. $\neg(A > B) > (\neg(\neg A > C) > \bot)$ (2,10, Consequent Agglomeration)

Recall:

Triviality 2. $\neg(A > C) > (B > \neg C)$

Proof.

1. $C > (A > C)$ (Persistence)
2. $\neg(A > C) > (C > (A > C))$ (1, Lemma 1)
3. $\neg(A > C) > (C > \neg(A > C))$ (Persistence)
4. $\neg(A > C) > (C > \bot)$ (Consequent Agglomeration, 3,4)
5. $\neg(A > C) > (B > \neg C)$ (MOD, 4)

Next we show that, given the previous background logic plus the principle of Contraction, we can prove:

Triviality 3. $\neg(A > C) > \neg(A \supset C)$

Proof:

1. $\neg(A > C) > (\neg(A > C) > \neg C)$ (Triviality 1)

2. $\neg(A > C) > \neg C$ (1, Contraction)

3. $\neg(A > C) > (\neg(\neg A > \bot) > \bot)$ (Triviality 2)

4. $\neg(A > C) > (\neg(A > C) > (\neg A > \bot))$ (3, MOD, Consequent Agglomeration)

5. $\neg(A > C) > (\neg A > \bot)$ (4, Contraction)

6. $\neg(A > C) > (\neg(A > C) > A)$ (5, MOD, Consequent Agglomeration)

7. $\neg(A > C) > A$ (6, Contraction)

8. $\neg(A > C) > \neg(A \supset C)$ (2,7, Consequent Agglomeration)

Triviality 4. $(A \vee B) > B, (B \vee C) > B \vdash (A \vee B) > \neg B > \bot$, for pairwise inconsistent Boolean A, B, C, given Identity, Consequent Agglomeration, CSO and Boolean Persistence.

Proof.

1. $(A \vee B) > B$ (assumption)

2. $(B \vee C) > B$ (assumption)

3. $B > (A \vee B)$ (Identity, Consequent Closure)

4. $B > (B \vee C)$ (Identity, Consequent Closure)

5. $(A \vee B) > (B \vee C)$ (1,3,4, CSO)

6. $(B \vee C) > (A \vee B)$ (2,3,4, CSO)

7. $(A \vee B) > (\neg B > (A \vee B))$ (Boolean Persistence)

8. $(A \vee B) > (\neg B > \neg B)$ (Identity, Consequent Closure)

9. $(A \vee B) > (\neg B > A)$ (7,8, Consequent Agglomeration)

10. $(B \vee C) > (\neg B > (B \vee C))$ (Boolean Persistence)

11. $(B \vee C) > (\neg B > \neg B)$ (Identity, Consequent Closure)

12. $(B \vee C) > (\neg B > C)$ (10, 11, Consequent Agglomeration)

13. $(A \vee B) > (\neg B > C)$ (5,6,12, CSO)

14. $(A \vee B) > (\neg B > \bot)$ (9, 13, Consequent Agglomeration)

C Soundness of A

We prove that A is sound on basic informational variably strict frames.

Fact 1. The axioms of PC are valid and the rules preserve validity.
Proof. Routine.

Fact 2. LLE preserves validity.
Proof. By the definition of validity, if $\models A \leftrightarrow B$ then A and B have the same truth-value at all proper points. Thus the maximal proper subset of $[\![A]\!]$ is the maximal proper subset of $[\![B]\!]$. So by Proper Inputs, for any C at any $\langle w, i\rangle$, $\langle w, i\rangle \models A > C \leftrightarrow B > C$.

Fact 3. Consequent Agglomeration preserves validity.
Proof. Suppose $B_1, ..., B_n \models C$ and that $\langle w, i\rangle \models A > B_1$, ... and $A > B_n$. By Propriety, if $\langle w', i'\rangle \in f([\![A]\!], \langle w, i\rangle)$ then $\langle w', i'\rangle$ is proper. So if $\langle w', i'\rangle \models B_1$, ..., and B_n, $\langle w', i'\rangle \models C$.

Fact 4. Identity is valid.
Proof. First recall that $i \models A$ iff for all $w \in i$ $\langle w, i\rangle \models A$. This is trivial when $f([\![A]\!], \langle w, i\rangle)$ is empty; so assume otherwise. The Update Constraint tells us that if $\langle w', i'\rangle \in f([\![A]\!], \langle w, i\rangle)$ then $i' \models A$. Moreover, by Propriety $\langle w', i'\rangle$ is proper. Thus $\langle w', i'\rangle \models A$.

Fact 5. When A is Boolean, $i + A = i \cap [\![A]\!]^i$, where $[\![A]\!]^i = \{w : \langle w, i\rangle \in [\![A]\!]\}$.
Proof. Routine induction on complexity.

Say that $\langle w, i\rangle \models \Box A$ iff for all $w' \in i$ $\langle w, i'\rangle \models A$; and define $\Diamond A$ as $\neg \Box \neg A$.

Fact 6. $\neg(A > C)$ is not persistent for Boolean A and C.
Proof. Note that $i \models \neg(A > C)$ only if $i \models \Diamond A$. Otherwise $i + A$ contains only the empty set and thus by the Update Constraint, $f([\![A]\!], \langle w, i\rangle)$ would be empty for all $w \in i$. But now since $\Diamond A$ is clearly not persistent, neither is $\neg(A > C)$.

Fact 7. Persistence is invalid.
Proof. $\langle w, i\rangle \not\models \neg(p > q) > (\neg p > \neg(p > q))$, when i contains some p-worlds. $\neg(p > q) > (\neg p > \neg(p > q))$ holds at $\langle w, i\rangle$ iff for all $\langle w', i'\rangle \in f([\![\neg(p > q)]\!], \langle w, i\rangle)$ for all $\langle w'', i''\rangle \in f([\![p]\!], \langle w', i'\rangle$ $\langle w'', i''\rangle \models \neg(p > q)$. $\langle w'', i''\rangle \in f([\![p]\!], \langle w', i'\rangle$ $\langle w'', i''\rangle \models \neg(p > q)$ iff for some $\langle w''', i'''\rangle \in f([\![p]\!], \langle w'', i''\rangle)$ $\langle w''', i'''\rangle \models \neg q$. But note that $i'' = i' + \neg p$ and $i''' = i'' + p$ so i''' is empty. So $f([\![p]\!], \langle w'', i''\rangle)$ is in fact empty.

Fact 8. Boolean Persistence is valid.

Proof. Recall that A is persistent iff when $i \models A$ and $i' \subseteq A$ then $i' \models A$. $A > (C > A)$ holds at $\langle w, i \rangle$ iff for all $\langle w', i' \rangle \in f(\llbracket A \rrbracket, \langle w, i \rangle)$: for all $\langle w'', i'' \rangle \in f(\llbracket C \rrbracket, \langle w', i' \rangle)$ $\langle w'', i'' \rangle \models A$. Pick an arbitrary $\langle w', i' \rangle \in f(\llbracket A \rrbracket, \langle w, i \rangle)$ and an arbitrary $\langle w'', i'' \rangle \in f(\llbracket C \rrbracket, \langle w', i' \rangle)$ $\langle w'', i'' \rangle \models A$. First note that $i' \in i + A$ and so $i' \models A$. Now $i'' = i' + C$. Since A is persistent $i'' \models A$ also. Since $\langle w'', i'' \rangle \in P$, $w'' \in i''$. So $\langle w'', i'' \rangle \models A$.

Fact 7. Boolean CSO is valid.

Proof. Suppose that A, B, C are Boolean and that $A > B$, $B > A$, and $A > C$ hold at $\langle w, i \rangle$. Thus $\downarrow f(\llbracket A \rrbracket, \langle w, i \rangle) \subseteq \llbracket B \rrbracket^i$ and $\downarrow f(\llbracket B \rrbracket, \langle w, i \rangle) \subseteq \llbracket A \rrbracket^i$. By Reciprocity $\downarrow f(\llbracket A \rrbracket, \langle w, i \rangle) = \downarrow f(\llbracket B \rrbracket, \langle w, i \rangle)$. Since C is Boolean, for all $\langle w', i' \rangle \in f(\llbracket A \rrbracket, \langle w, i \rangle)$: $\langle w', i' \rangle \models C$ iff $\langle w', i'' \rangle \in f(\llbracket B \rrbracket, \langle w, i \rangle)$: $\langle w', i'' \rangle \models C$.

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